Estimating Volatilities And Correlations for Risk Management

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ESTIMATING VOLATILITY:

- The continuously compounded return over successive days is:

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \text{ where } S_i \text{ is asset price at time i}$$

- The mean return of these individual returns is calculated as:

$$\bar{u} = \frac{1}{m}\sum_{i=1}^{m} u_{n-i} \quad \text{where m is the number of oservations leading up to the present period}$$

- Maximum likelihood estimator of variance, when mean return is very small:

$$\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} u_{n-i}^2$$

- If we weight recent data more heavily, than historical data then the current level of volatility calculated as:

$$\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} \alpha_i u_{n-i}^2 \quad \text{where } \alpha_i \text{ is the weight on the return 1 day ago}$$

- ARCH(m) can be represented as:

$$\sigma_n^2 = \gamma V_L + \sum_{i=1}^{m} \alpha_i u_{n-i}^2 \text{ with } \gamma + \sum \alpha_i = 1$$

$$\sigma_n^2 = \omega + \sum_{i=1}^{m} \alpha_i u_{n-i}^2 \quad \text{where } \omega = \gamma V_L (\text{long run variance weight})$$

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THE EXPONENTIALLY WEIGHTED MOVING AVERAGE(EWMA) MODEL:

- The **exponentially weighted moving average** (EWMA) model :

$$\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2 \qquad \text{where } \lambda = \text{weight on previous volatility estimate}$$

Example: EWMA model

The decay factor in an exponentially weighted moving average model is estimated to be 0.94 for daily data. Daily volatility is estimated to be 1%, and today's stock market returns is 2%. What is the new estimated of volatility using the EWMA model?

➢ THE GARCH(1,1) MODEL:

❑The **generalized autoregressive conditional heteroskedastic** (GARCH) (1,1) model:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

$where: \alpha =$ weighting on the previous period's return,
$\beta =$ weighting on the previous volatility estimate,
$\omega =$ weighted long run variance $= \gamma V_L$,
$V_L =$ long run average variance $= \frac{\omega}{1-\alpha-\beta}$
and $\alpha + \beta + \gamma = 1$, also $\alpha + \beta <$
1 for stability so that γ is not negative

Example: GARCH(1,1) model

The parameters of a generalized auto-regressive conditional heteroskedastic (GARCH)(1,1) model are w =0.000003, $\alpha = 0.04$, and $\beta = 0.92$. if daily volatility is estimated to be 1%, and today's stock market return is 2%, what is the new estimated of volatility using the GARCH(1,1) model, and what is the implied long-run volatility level?

➢ THE GARCH(1,1) MODEL:

Mean Reversion

- Empirical data indicates that volatility exhibits a mean-reverting characteristic.
- GARCH model tends to display a better theoretical justification than the EWMA model.
- In GARCH model the sum of the weights of α and β are sometimes greater than, which causes instability in the volatility estimation.
- The sum α and β is called the persistence.
- The persistence describes the rate at which the volatility will revert to its long term value following a large movement.
- The higher the persistence (given that is less than one), the longer it will take to revert to the mean following a shock or large movement.
- A persistence of one means that there is no reversion and with each change in volatility a new level is attained.

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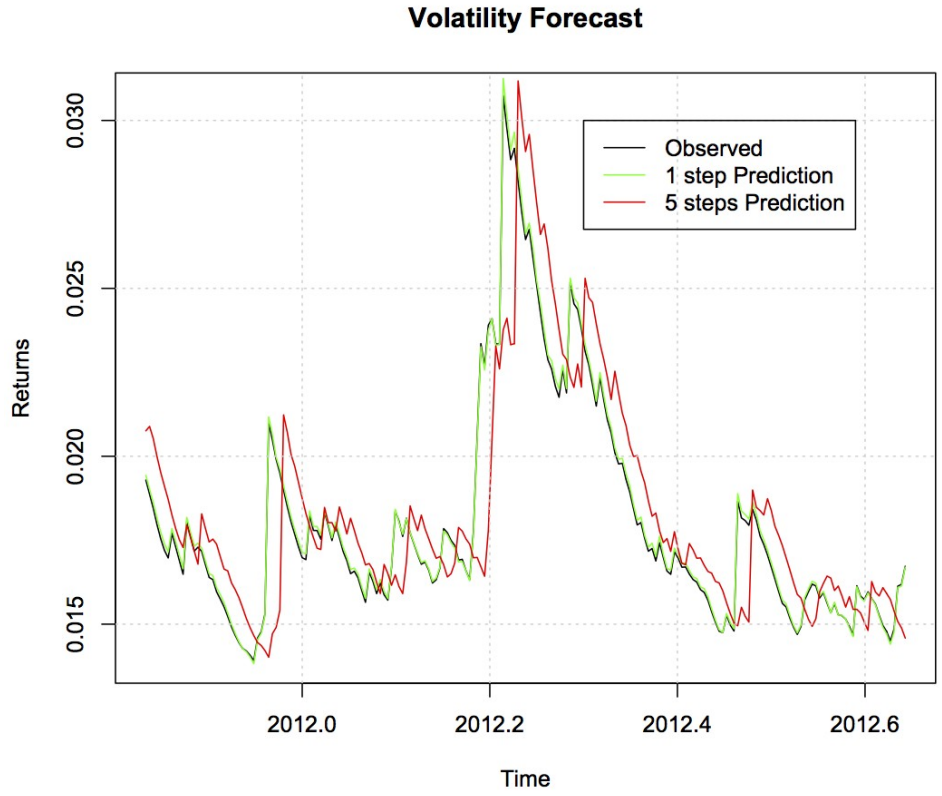
ESTIMATION AND PERFORMANCE OF GARCH MODELS:

- Maximum likelihood estimators select values of model parameters that maximize the likelihood that the observed data will occur in a sample.
- GARCH models are estimated using maximum like hood techniques.
- The estimation process:
  - Guess of the model's parameters.
  - Then a calculation of the like hood function based on those parameter estimate is made.
  - The parameters are then slightly adjusted until the like hood function fails to increase, at which time the estimation process assumes it has maximized the function and stops.
  - The values of the parameters at the point of maximum value in the like hood function are then used to estimate GARCH model volatility.
- GARCH models do a fine job at forecasting volatility from a volatility term structure perspective.



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