Modeling Cycles MA, AR, And ARMA Model

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FIRST-ORDER MOVING AVERAGE PROCESS:

- A moving average process is a linear regression of the current values of a time series against both the current and previous unobserved white noise error terms, which are random shocks.
- The first-order moving average [MA(1)] process has a mean of zero and a constant variance and can be defined as:

$$y_t = \varepsilon_t + \theta\varepsilon_{t-1}$$

- Where:
  - y_t = the time series variable being estimated
  - ε_t =current random white noise shock
  - ε_{t-1}=one-period lagged random white noise shock
  - θ =coefficient for the lagged random shock
- Current value is a function of current and lagged unobservable shocks.
- Each shock has impact over two periods: contemporaneous impact and one-period delayed impact
- The MA(1) coefficient θ controls the degree of serial correlation. It may be positive or negative.

- Example: Using daily demand for ice cream (y_t) represented as: $y_t = \varepsilon_t + 0.3\varepsilon_{t-1}$
  - In this equation, the error term (ε_t) is the daily change in temperature. Using only the current period's error term (ε_t), if the daily change in temperature is positive, then we would estimate that daily demand for ice cream would also positive. But, if the daily change yesterday (ε_{t-1}) was also positive, then we would expect an amplified impact on our daily demand for ice cream by a factor of 0.3.

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FIRST-ORDER MOVING AVERAGE PROCESS:

- Auto-correlation (p) cutoff can be calculated as:

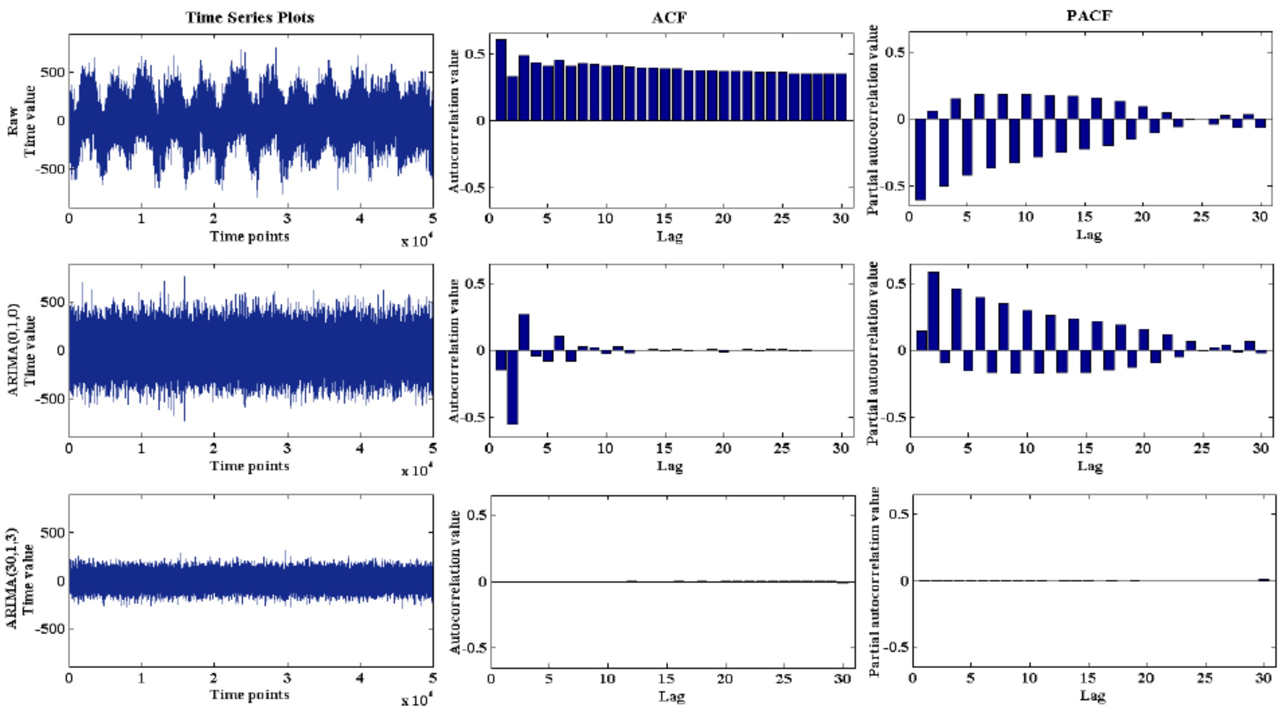
$$\rho_1 = \frac{\theta_1}{1-\theta_1^2}\text{; where } \rho_\tau = 0 \text{ for } \tau > 1$$

- For any value beyond the first lagged error term, the auto-correlation will be 0 in an MA(1) process.

- Inversion of MA(1): also called autoregressive representation is expressed as:

$$\varepsilon_t = y_t - \theta\varepsilon_{t-1}$$

- The purpose is to calculate current shock from lagged shock & a lagged value.



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MA(q) PROCESS:

❖The MA(q) process is expressed in the following formula:

$$y_t = \theta_1 + \theta_t \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$

Where:

y_t =the time series variable being estimated

ε_t =current random white noise shock

ε_{t-1}=one-periods lagged random white noise shock

ε_{t-q}=qth period lagged random white noise shock

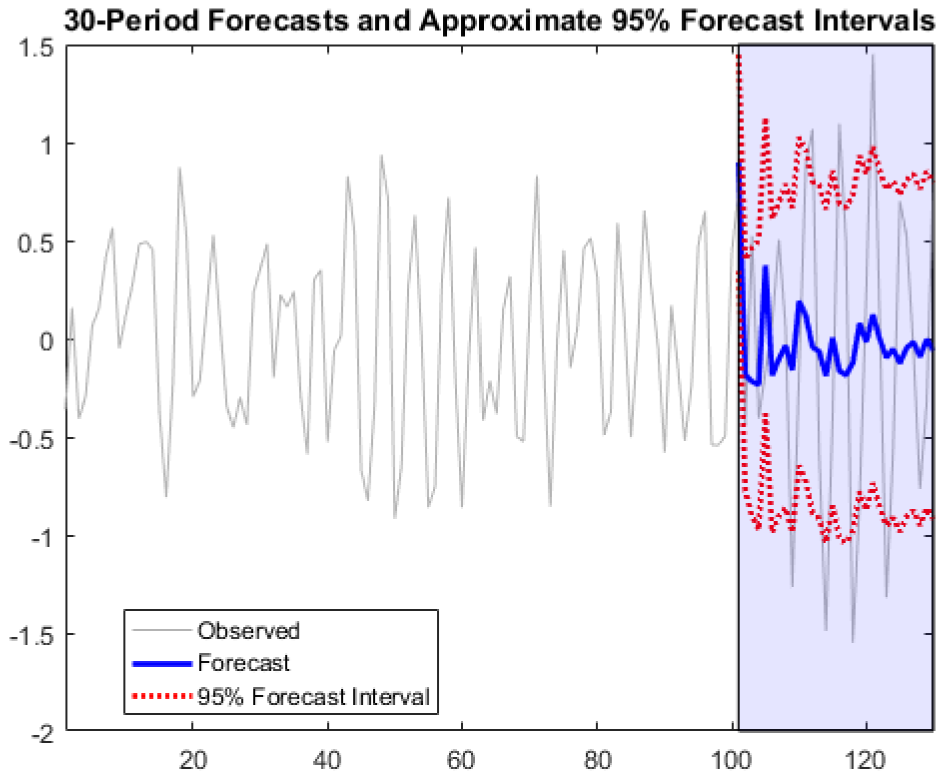
θ =coefficients for the lagged random shocks

❖The MA(q) process theoretically captures complex patterns in greater detail, which can potentially provide for more robust forecasting.

❖This also lengthens the memory from one period to the qth period

❖The first q autocorrelations of MA(q) are non-zero, the autocorrelations above q are zero.

❖MA(q) may be good approximation of MA(∞).



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- **FIRST-ORDER AUTOREGRESSIVE PROCESS:**

❖The first-order auto-regressive [AR(1)]process must also have a mean of zero and a constant variance.
❖It is specified in the from of a variable regressed against itself in a lagged form. This relationship can be shown in the following formula: $y_t = \emptyset y_{t-1} + \varepsilon_t$
Where:

y_t =the time series variable being estimated
ε_t =current random white noise shock
y_{t-1}=one-periods lagged observation of the variable being estimated
ϕ =coefficients for the lagged random shocks

❖In order for an AR(1) process to be covariant stationery the absolute value of the coefficient on the lagged operator must be less than one (i.e., | ϕ | <1).
❖The Yule-Walker equation is used to estimate the auto-regressive parameters.

$$\rho_t = \emptyset^t \, for \; t = 0,1,2, \ldots.$$

❖If | ϕ | < 1, this is a general linear process with geometrically declining coefficients.
❖The impact of a shock becomes smaller and smaller as time passes.
❖An AR(1) with | ϕ | = 1 is known as a random walk or unit root process. It is a non-stationary process.

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- **AR(p) PROCESS:**

❖The AR(p) process expands the AR(1) process out to the p[th] observation:

$$y_t = \emptyset_t y_{t-1} + \emptyset_t y_{t-2} + \cdots + \emptyset_p y_{t-p} + \varepsilon_t$$

Where:

y_t	=the time series variable being estimated
y_{t-1}	=one-period lagged observation of the variable being estimated
y_{t-p}	=pth period lagged observation of the variable being estimated
ε_t	=current random white noise shock
ϕ	=coefficients for the lagged observation sof the variable being estimated

❖The AR(p) process is also covariance stationary if $|\phi| < 1$ and it exhibits the same decay in auto-correlations that was found in the AR(1) process.

❖However, while an AR(1) process only evidences oscillation in its auto-correlations (switching from positive to negative) when the coefficient is negative, an AR(p) process will naturally oscillate as it has multiple coefficients interacting with each other.

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- **AUTOREGRESSIVE MOVING AVERAGE PROCESS:**

❖Autoregressive moving average (ARMA) process expressed as:

$$y_t = \emptyset y_{t-1} + \varepsilon_t + \emptyset \varepsilon_{t-1}$$

Where:

y_t=the time series variable being estimated

ϕ=coefficient for the lagged observation of the variable being estimated

y_{t-1}=one-period lagged observation of the variable being estimate

ε_t =current random white noise shock

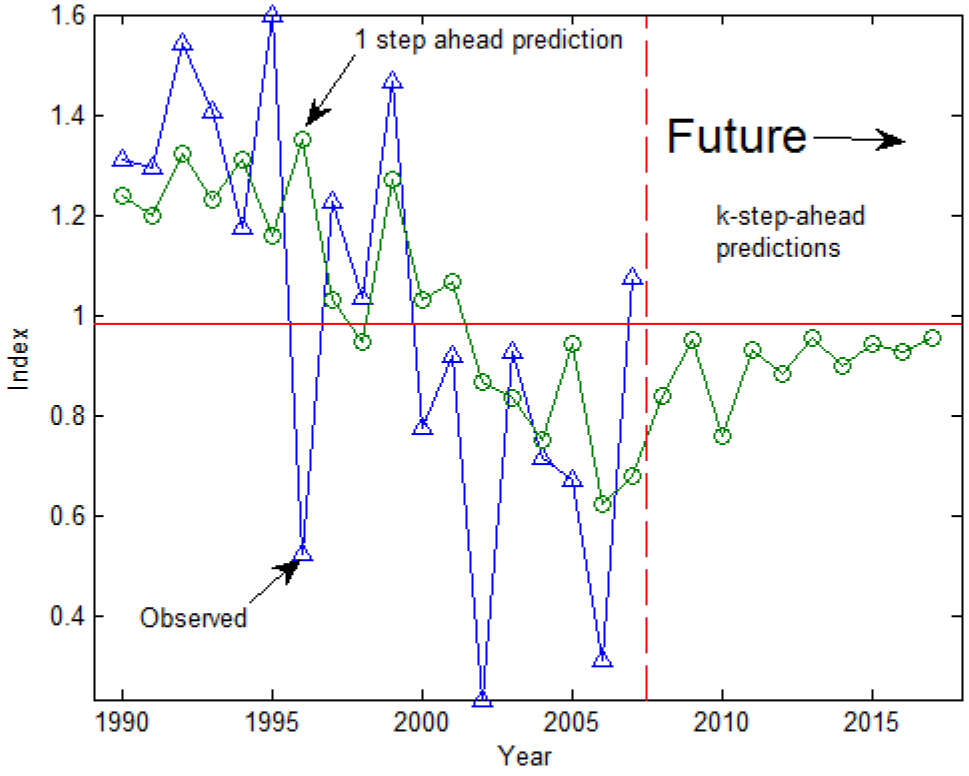
θ= coefficient for the lagged random shocks

ε_{t-1} =one-period lagged random white noise shock

❖In order for the ARMA process to be covariance stationary $|\theta|< 1$.

❖The auto-correlations in, an ARMA process will also decay gradually.

❖Example: ***an ARMA (3,1) model mean 3 lagged operators in the AR portion of the formula and 1 lagged operator on the MA portion.***



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➢ **APPLICATION OF AR AND ARMA PROCESS:**

❖If the autocorrelations decay gradually, the analyst should use either an auto-regressive (AR) process or an autoregressive(ARMA) process.

❖If there is some pattern of seasonality, then ARMA model is preferred.

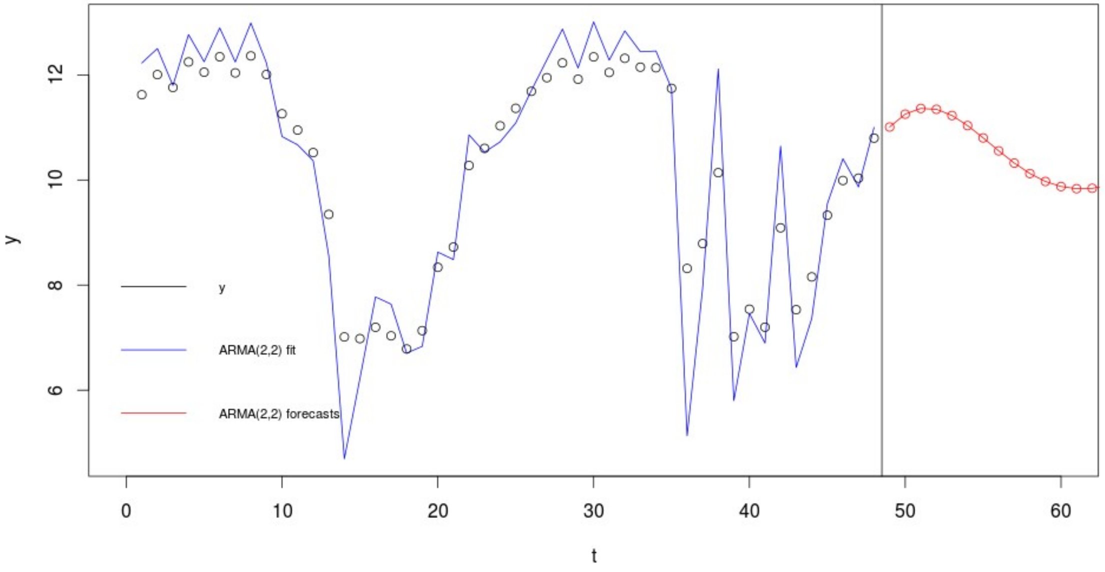
❖Example:

❖ Base AR(2) model that adds in a constant value (μ) if all other values are zero:

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t$$

❖An ARMA (3,1) model

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \phi \varepsilon_{t-1} + \varepsilon_t$$



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