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Regression with a Single Regressor: Hypothesis Tests and Confidence Intervals

Regression with a Single Regressor

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- **REGRESSION COEFFICIENT CONFIDENCE INTERVALS:**
  - Hypothesis testing for a regression coefficient is used to test whether an estimated slope coefficient is statistically different from zero.
  - The test for the regression coefficient is null hypothesis H_0: $B_1 = 0$ and the alternative hypothesis is H_A : $B_1 \neq 0$.
  - The confidence interval for the regression coefficient, B_1, is calculated as;

$$b_1 \pm (t_c \times s_{b_1})$$

where degree of freedom = n-2

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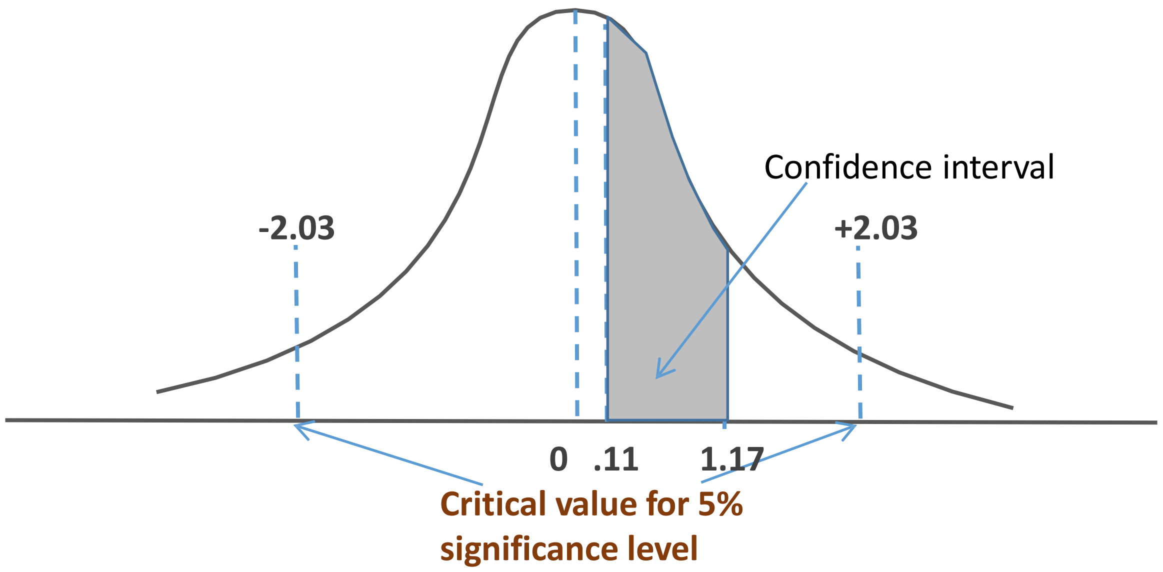
REGRESSION COEFFICIENT CONFIDENCE INTERVALS:

Example: Calculating the confidence interval for a regression coefficient.

The estimate slope coefficient, B_1, from a regression run on WPO stock is 0.64 with a standard error equal to 0.26. assuming that the sample had 36 observations. Calculate the 95% confidence interval for B_1.



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REGRESSION COEFFICIENT HYPOTHESIS TESTING :

- A t-test may also be used to the hypothesis that the true slope coefficient, B_1, is equal to some hypothesized value. Letting b_1 be the point estimate for B_1 the appropriate test statistic with n – 2 degrees of freedom is:

$$t = \frac{b_1 - B_1}{s_{b1}}$$

- The decision rule for tests of significance for regression coefficients is:
- Reject H_0 if $t > + t_{criitical}$ or $t < -t_{critical}$

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➢ REGRESSION COEFFICIENT HYPOTHESIS TESTING :

Example: Hypothesis test for significance coefficients

Again, suppose that the estimated slope coefficient for the WPO regression is 0.64 with a standard error equal to 0.26. assuming that the sample has 36 observations, determine if the estimated slope coefficient is significantly different than zero at a 5% level of significance.

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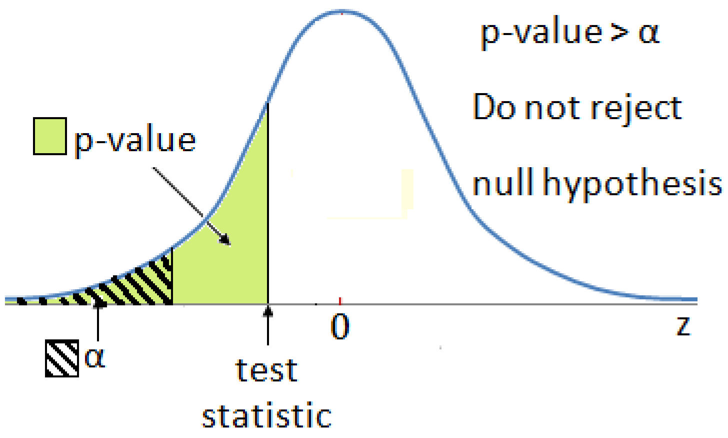
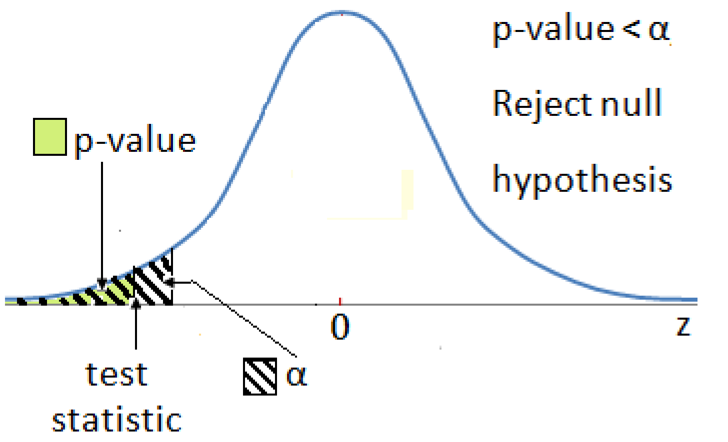
➢ **REGRESSION COEFFICIENT HYPOTHESIS TESTING :**

❑**The p –values**

For two-tailed test, the p-value is the probability that lies above the positive value of the computed test statistic plus the probability that lies below the negative value of the computed test statistic.





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➢ **PREDICTED VALUES:**

- ❑Predicted values are values of the dependent variable based on the estimated regression coefficients and a prediction about the value of the independent variable.
- ❑They are the values that are predicated by the regression equation, given an estimate of the independent variable.
- ❑For a simple regression, the predicted (or forecast) value of Y is: $\hat{Y} = b_0 + b_1 X_p$

Where:

$\hat{Y}$ =predicted value of the dependent variable

X_P = forecasted value of the independent variable

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➢ **PREDICTED VALUES:**

Example: Predicting the dependent variable

Given the regression equation: $\widehat{\text{WPO}} = -2.3\% + (0.64)(\widehat{\text{S\&P500}})$

Calculate the predicated value of WPO excess returns if forecasted S & P 500 excess returns are 10%.

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- **CONFIDENCE INTERVALS FOR PREDICTED VALUES:**

The equation for the confidence interval for a predicted value of Y is:

$$\hat{Y} \pm (t_c \text{ x } s_f) \Rightarrow \left[\hat{Y} - (t_c \text{ x } s_f) < Y < \hat{Y} + (t_c \text{ x } s_f)\right]$$

Where: t_c = two tailed critical t-values at the desired level of significance with df=n-2

s_f = standard error of the forecast

the standard error of the forecast can be calculated as:

$$s_f^2 = SER^2 \left[1 + \frac{1}{n} + \frac{(X - \bar{X})^2}{(n-1)s_x^2}\right.$$

Where:

SER^2 =variance of the residual s= the square of the standard error of the regression

S_x^2 =variance of the independent variable

X =value of the independent variable for which the forecast was made.

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- **CONFIDENCE INTERVALS FOR PREDICTED VALUES:**

Example: Confidence interval for a predicted value

Calculate a 95% prediction interval on the predicted value of WPO from the previous example. Assume the standard error of the forecast is 3.67, and the forecasted value of S & P 500 returns is 10%

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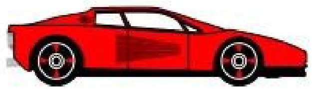
- **DUMMY VARIABLES:**
  - ❑Dummy variables are assigned a value of "0" or "1."
  - ❑Dummy variables and are often used to quantify the impact of qualitative events.
  - ❑The estimate regression coefficient for dummy variable indicated the difference in the dependent variable for the category represented by the dummy variable and the average value of the dependent variable for all classes except the dummy variable class.

$$y_t = \beta_1 + \beta_2 X_t + \beta_3 D_t + e_t$$

y_t = speed of car in miles per hour

X_t = age of car in years



$D_t = 1$ if **red** car, $D_t = 0$ otherwise.

Police: **red** cars travel faster.

$H_0: \beta_3 = 0$

$H_1: \beta_3 > 0$

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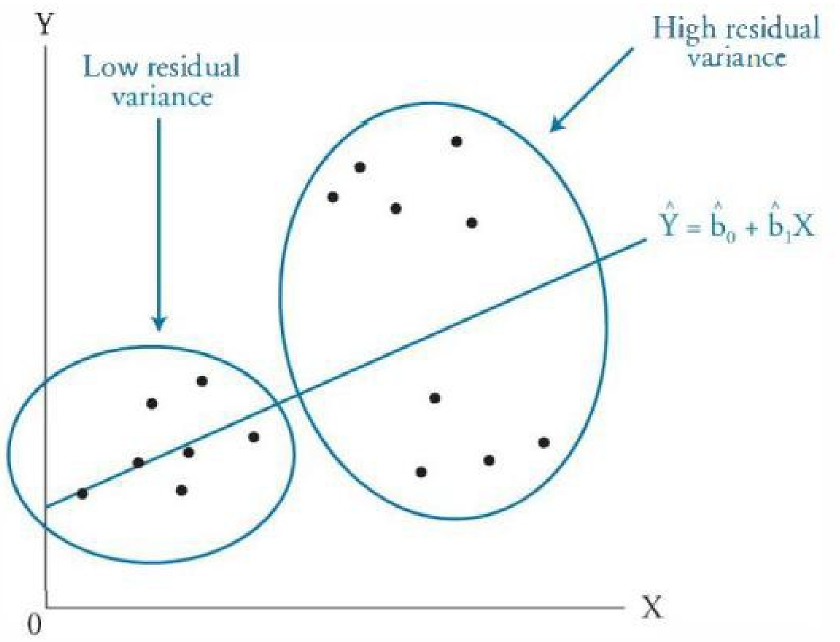
- **HETEROSKEDASTICITY :**

  - ❑If the variance of the residuals is constant across all observations in the sample, the regression is said to be homoskedastic.
  - ❑When the variance of the residuals is not the same across all observations in the sample, the regression exhibits heteroskedasticity.
  - ❑When the heteroskedasticity is not related to the level of the independent variables, then it is called unconditional heteroskedasticity.
  - ❑When the heteroskedasticity is related to the level of the independent variables, then it is called conditional heteroskedasticity.



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➢ HETEROSKEDASTICITY :

Effect of Heteroskedasticity on Regression Analysis

- The standard errors are usually unreliable estimates.
- The coefficient estimates (the b_j) aren't affected.

Detecting Heteroskedasticity

A scatter plot of the residuals versus one of the independent variable can reveal patterns among observations.

Correcting Heteroskedasticity

- The most common method is to calculate robust standard errors.
- These robust standard errors are used to recalculate the t-statistic using the original regression coefficients.

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➢ THE GAUSS-MARKOV THEOREM :

❑The Gauss-Markov theorem says that if the linear regression model assumptions are true and the regression errors display homoskedasticity, then the OLS estimators have the following properties.

- ▪The OLS estimated coefficients have the minimum variance compared to other methods of estimating the coefficients.
- ▪The OLS estimated coefficient is based on linear functions.
- ▪The OLS estimated coefficients are unbiased.
- ▪The OLS estimate of the variance of the errors is unbiased.
- ▪ The OLS estimate is also called "BLUE" → best linear unbiased estimators

❑Limitation of the Gauss-Markov theorem:

- ▪Its conditions may not hold in practice, particularly when the error terms are heteroskedastic.
- ▪Alternative estimators, which are not linear or unbiased, may be more efficient than OLS estimators.

Examples of these alternative estimators include:

- •the weighted least squares estimator
- •the least absolute deviations estimator

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Thanks

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