/02

Bayesian Analysis

Objective

- Describe Bayes' theorem and apply this theorem in the calculation of conditional probabilities.
- Compare the Bayesian approach to the frequentist approach.
- Apply Bayes' theorem to scenarios with more than two possible outcomes and calculate posterior probabilities.

Bayesian Analysis

➢ BAYES' THEOREM:

➢Bayes' theorem for two random variables A and B is defined as follows:

Likelihood
How probable is the evidence given that our hypothesis is true?

Prior
How probable was our hypothesis before observing the evidence?

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

Posterior
How probable is our hypothesis given the observed evidence?

Marginal
How probable is the new evidence under all possible hypothesis?

Bayesian Analysis

➢ BAYES' THEOREM:

➢The joint probability of both events A and B occurring can be determined by the following two equations. Notice that it does not matter which event occurred first. The first equation is used if event B occurred first and the second equation is used if event A occurred first.

$$P(AB) = P(A|B) \times P(B)$$

$$P(AB) = P(B|A) \times P(A)$$

➢Regardless of which unconditional event occurred first, the joint probability of both occurring is the same. Thus, these two equations can be combined.

➢Bayes' theorem provides a framework for determining the probability of event occurring given that another random event has already has already occurred.

Bayesian Analysis

➢ BAYES' THEOREM:

➢Example:

Suppose a bond manager is interested in knowing the probability of Bond A defaulting given that Bond B is already in default. Table below is a probability matrix defining two events for both bonds, default and no default.

		Bond A		
		No Default	Default	
Bond B	No Default	80%	8%	88%
	Default	8%	4%	12%
		88%	12%	100%

The recent financial crisis beginning in 2007 illustrated that bond default are highly correlated. The conditional probability of Bond A defaulting given that Bond B is already in default is defined by:

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{4\%}{12\%} = \frac{1}{3} or\ 33.3333\%$$

Bayesian Analysis

➢ BAYES' THEOREM:

Example : Bayes' theorem (1)

Suppose you are an equity analyst for ABC Insurance Company. You manage an equity fund of funds and use historical data to categorize the managers as excellent or average. Excellent managers are expected to outperform the market 70% of the time. Average managers are expected to outperform the market only 50% of the time. Assume that the probabilities of managers outperforming the market for any given year are independent of their performance in prior years. ABC Insurance Company has found that only 20% of all fund managers are excellent managers and the remaining 80% are average mangers.

A new fund manager to the portfolio started three years ago and outperformed the market all three years. What is the probability that the new manager was an excellent manager when she first started managing portfolios three years ago?

Bayesian Analysis

➢ BAYES' THEOREM:

Example: Bayes' theorem (2)

Using the same information given in the previous example, what are the probabilities that the new manager is an excellent or average manager today?

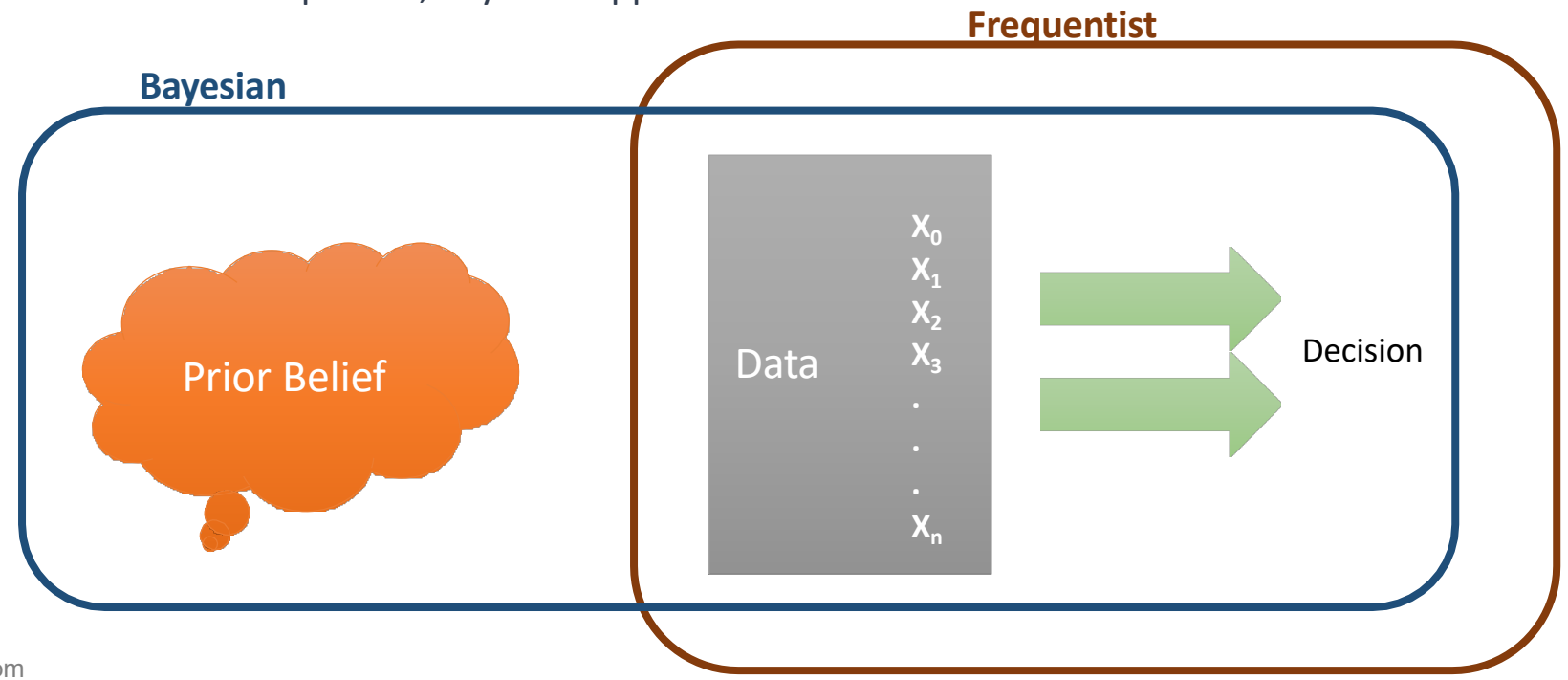
Example: Bayes' theorem (3)

Using the same information given in the previous two examples, what is the probability that the new manager will beat the market next year, given that the new manger outperformed the market the last three years?

Bayesian Analysis

➢ BAYESIAN APPROACH VS. FREQUENTIST APPROACH:

- ➢The frequentist approach involves drawing conclusions from sample data based on the frequency of that data.
- ➢Parameters are considered fixed but unknown.
- ➢We can't assign a distribution.
- ➢Bayesian approach is instead based on a prior belief regarding the probability of an event occurring.
- ➢Parameters are considered random & unknown.
- ➢Frequentist approach is more useful in large sample size.
- ➢With small sample size, Bayesian approach is more useful.



Bayesian Analysis

➢ BAYES' THEOREM WITH MULTIPLE STATES:

➢Example: Bayes' theorem with three outcomes

The prior belief regarding the probabilities of a manager outperforming the market are 80% for an excellent manager, 50% for an average manager, and 20% for a below average manager. Furthermore, there is a 15% probability that a manager is excellent, a 55% probability that a manager is average, and a 30% probability that a manager is below average. These probabilities of manager performance are noted as follows:

P(p = 0.8) = 15%

P(p = 0.5) = 55%

P(p = 0.2) = 30%

Suppose a new fund manager outperforms the market two years in a row. Given the manager performance probabilities above, how is Bayesian analysis applied to updated prior expectations regarding the new manger's ability?

Thanks
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