/02 Basic Statistics

Objective

- Interpret and apply the mean, standard deviation, and variance of a random variable.
- Calculate the mean, standard deviation, and variance of a discrete random variable.
- Interpret and calculate the expected value of a discrete random variable.
- Calculate and interpret the covariance and correlation between two random variables.
- Calculate the mean and variance of sums of variables.
- Describe the four central moments of a statistical variable or distribution: mean, variance, skewness, and kurtosis.
- Interpret the skewness and kurtosis of a statistical distribution, and interpret the concepts of coskewness and cokurtosis.
- Describe and interpret the best linear unbiased estimator.

Basic Statistics

➢ Introduction:

➢The word statistic is used to refers to data and the methods we use to analyze data.
➢Descriptive statistics are used to summarize the important characteristics of large data sets.
➢Inferential statistic, pertain to the procedures used to make forecasts, estimates, or judgments about a large set of data on the basis of the statistical characteristics of a smaller set (a sample).
➢A population is defined as the set of all possible members of a stated group.
➢Measures of central tendency identify the center, or average, of a data set.

Population Mean $\mu = \frac{\sum_{i=1}^{N} X_i}{N}$

Sample Mean $\bar{x} = \frac{\sum_{i=1}^{N} X_i}{N}$

Median

Mode

Example:
Calculate Mean, Mode & Median from the following data set:
12%, 25%, 34%, 15%, 19%, 44%, 54%, 34%, 22%, 28%, 17%, 24%.

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➢ **Introduction:**

<u>**Geometric Mean of Returns**</u>

$$R_{GM} = \sqrt[n]{(1 + R_1)x(1 + R_2)x \dots x(1 + R_n)} - 1$$

Example: Geometric mean return

For the last three years, the returns for Acme Corporation common stock have been -9.34%, 23.45%, and 8.92%. compute the compound rate of return over the 3-year period.

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➢ **EXPECTATIONS:**

➢The expected value is the weighted average of the possible outcomes of a random variable where the weights are the probabilities that the outcomes will occurs.

$$E(X) = \mu = \sum P(x_i)x_i = P(x_1)x_1 + P(x_2)x_2 + \cdots + P(x_n)x_n$$

Example: Expected earnings per share

The probability distribution of EPS for Ron's Stores is given in the figure below. Calculate the expected earnings per share.

Probability	*Earnings Per Share*
10%	*1.80*
20%	*1.60*
40%	*1.20*
30%	*1.00*

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➢ EXPECTATIONS:

Properties of expectation include:

1. If c is any constant, then: $E(cX) = cE(X)$
2. If X and Y are any random variables, then: $E(X + Y) = E(X) + E(Y)$
3. If c and a are constant, then: $E(cX + a) = cE(X) + a$
4. If X and Y are independent random variables, then: $E(XY) = E(X) \times E(Y)$
5. If X and Y are NOT independent, then: $E(XY) \neq E(X) \times (EY)$
6. If X is a random variable, then: $E(X^2)$ $[E(X]^2$

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➢ **VARIANCE AND STANDARD DEVIATION :**

The mean and variance of a distribution are defined as the first and second moments of the distribution, respectively. Variance is defined as:

$$Var(X) = E[X - \mu]^2 = E(X^2) - [E(X)]^2$$

Standard deviation is defined as:

$$\sigma_X = \sqrt{E[X - \mu]^2} = \sqrt{E(X^2) - [E(X)]^2}$$

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VARIANCE AND STANDARD DEVIATION :

Properties of variance include:

1. If c is any constant, then: Var(c) = 0
2. If c is any constant, then: Var(cX) = c^2 x Var (X)
3. If c is any constant, then: Var (X+ c) = Var(X)
4. If a and c are constants, then: Var(aX + c) = a^2 x Var(X)
5. If X and Y are independent random variables, then:
    1. Var (X + Y) = Var(X) + Var(Y)
    2. Var (X – Y) = Var (X) + Var(Y)
6. If X and Y are independent and a and c are constants, then: Var(aX +cY) = a^2 x Var(X) + c^2 x Var(Y)

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➢ **VARIANCE AND STANDARD DEVIATION :**

Example: Computing variance and standard deviation

What is the variance and standard deviation of the sum of the points in tossing single coin if heads = 2 points and tails = 10 points?

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➢ VARIANCE AND STANDARD DEVIATION :

Proof that var(X) = E(X²) -E ²(X)

We have:

$$\text{var}(X) = E\left[(X-\mu)^2\right]$$

Expanding the bracket gives:

$$\text{var}(X) = E\left[X^2 - 2\mu X + \mu^2\right]$$

Splitting up the expectation gives:

$$\text{var}(X) = E(X^2) - E(2\mu X) + E(\mu^2)$$

Since μ is a constant, we can use $E(aX+b) = aE(X)+b$:

$$E(2\mu X) = 2\mu E(X)$$

$$E(\mu^2) = \mu^2$$

Hence:

$$\text{var}(X) = E(X^2) - 2\mu E(X) + \mu^2$$

But $\mu = E(X)$! So we get:

$$\text{var}(X) = E(X^2) - 2\mu^2 + \mu^2$$

$$= E(X^2) - \mu^2$$

$$= E(X^2) - E^2(X)$$

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➢ **VARIANCE AND STANDARD DEVIATION :**

Proof that var(aX+b) = a ² Var(X)

$$\text{var}(X) = E(X^2) - E^2(X)$$

Replacing X with $aX+b$ in our variance formula gives:

$$\text{var}(aX+b) = E[(aX+b)^2] - [E(aX+b)]^2$$

Expanding the brackets in the first term gives:

$$\text{var}(aX+b) = E[a^2X^2 + 2abX + b^2] - [E(aX+b)]^2$$

Splitting up the first expectation:

$$\text{var}(aX+b) = E(a^2X^2) + E(2abX) + E(b^2) - [E(aX+b)]^2$$

Using $E(aX+b) = aE(X)+b$ gives:

$$\text{var}(aX+b) = a^2E(X^2) + 2abE(X) + b^2 - [aE(X)+b]^2$$

Expanding the brackets in the last term and simplifying:

$$\text{var}(aX+b) = a^2E(X^2) + 2abE(X) + b^2 - [a^2E^2(X) + 2abE(X) + b^2]$$

$$= a^2E(X^2) - a^2E^2(X)$$

$$= a^2\left\{E(X^2) - E^2(X)\right\}$$

$$= a^2\,\text{var}(X)$$

Basic Statistics

➢ CONVARIANCE AND CORRELATION:

➢The variance and standard deviation measure the dispersion, or volatility, of only one variable.
➢However, covariance & correlation measures relationships between two variable.
➢Covariance is the expected value of the product of the product of the deviation of the two random variables from their respective expected values.

$$Cov(R_iR_j) = E(R_iR_j) - E(R_i)xE(R_j)$$

Properties of covariance include:

1. If X and Y are independent random variables, then: Cov(X,Y) = 0
2. The covariance of random variable X with itself is the variance of X.
    1. Cov(X,X) = Var(X)
3. If a, b, c, and d are constants, then: Cov(a + bX, c + dY) = b x d x Cov(X, Y)
4. If X and Y are NOT independent, then:
    1. Var(X Y) = Var(X) + Var(Y) + 2 x Cov(X, Y)
    2. Var(X –Y) = Var(X) + Var(Y) – 2 x Cov(X, Y)

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➢ CONVARIANCE AND CORRELATION:

Example: Covariance

Assume that the economy can be in three possible states (S) next year: boom, normal, or slow economic growth. An expert source has calculated that P(boom 0 = 0.30, P(normal) = 0.5, and P(slow) = 0.20. The return for Stock A, R_A, and Stock B, R_B, under each of the economic states are provided in the table below. What is the covariance of the returns for Stock A and Stock B?

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➢ **CONVARIANCE AND CORRELATION:**

➢The covariance is difficult to interpret, because it can take on extremely large values, ranging from negative to positive infinity, and, like the variance, these values are expressed in terms of squared units.
➢Correlation coefficient is easy to interpret, because the values lies between -1 & +1 & it is unit neutral.

$$\rho_{(R_i,R_j)} = \frac{Cov(R_i,R_j)}{\sigma_{R_i}\sigma_{R_j}}, which\ implies\ Cov(R_i,R_j) = \rho_{(R_i,R_j)}\sigma_{R_i}\sigma_{R_j}$$

Properties of correlation include:

1. If Corr = 1.0, the random variables have perfect positive correlation. This means that a movement in one random variable results in a proportional positive movement in the other relative to its mean.
2. In Corr = -1..0, the random variables have perfect negative correlation. This means that a movement in one random variable results in an exact opposite proportional movement in the other relative toots mean.
3. If Corr = 0, there is no linear relationship between the variables, indicating that prediction of Ri, cannot be made on the basis of Rj using linear methods.

Basic Statistics

➢ MOMENTS AND CENTRAL MOMENTS:

- ➢The shape of a probability distribution can be described by the "moments" of the distribution.
- ➢Raw moments are measured relative to an expected value raised to the appropriate power.
- ➢The first raw moment is the mean of the distribution, which is the expected value of return:

$$E(R_i) = \mu_i = \sum_{i=1}^{n} P_i R_i^1$$

- ➢Generalizing, the kth raw moment is the expected value of Rk:

$$E(R^k) = \sum_{i=1}^{n} P_i R_i^k$$

- ➢Central moments are measured relative to the mean (i.e., central around the mean). The k th central moment is defined as:

$$E(R-\mu)^k = \sum_{i=1}^{n} P_i (R_i - \mu)^k$$

- ➢The second central moment is the variance of the distribution, which measures the dispersion of data.

$$Variance = \sigma^2 = E[R-\mu)^2$$

Basic Statistics

➢ MOMENTS AND CENTRAL MOMENTS:

➢The third central moment measures the departure from symmetry in the distribution. This moment will equal zero for a symmetric distribution (such as the normal distribution).

$$Third\ Central\ Moment = E[(R-\mu)^3]$$

➢The skewness statistic is the standardized third central moment.

$$Skewness = \frac{E([R-\mu)^3}{\sigma^3}$$

➢The fourth central moment measures the degree of clustering in the distribution

$$Fourth\ Central\ Moment = E[(R-\mu)^4]$$

➢The kurtosis statistic is the standardized fourth central moment of the distribution. Kurtosis refers to the degree of peakedness or clustering in the data distribution and is calculated as:

$$Kurtosis = \frac{E([R-\mu)^4}{\sigma^4}$$

➢Kurtosis for the normal distribution equal 3. Therefore, the excess kurtosis for any distribution equals:

Excess kurtosis = kurtosis – 3.

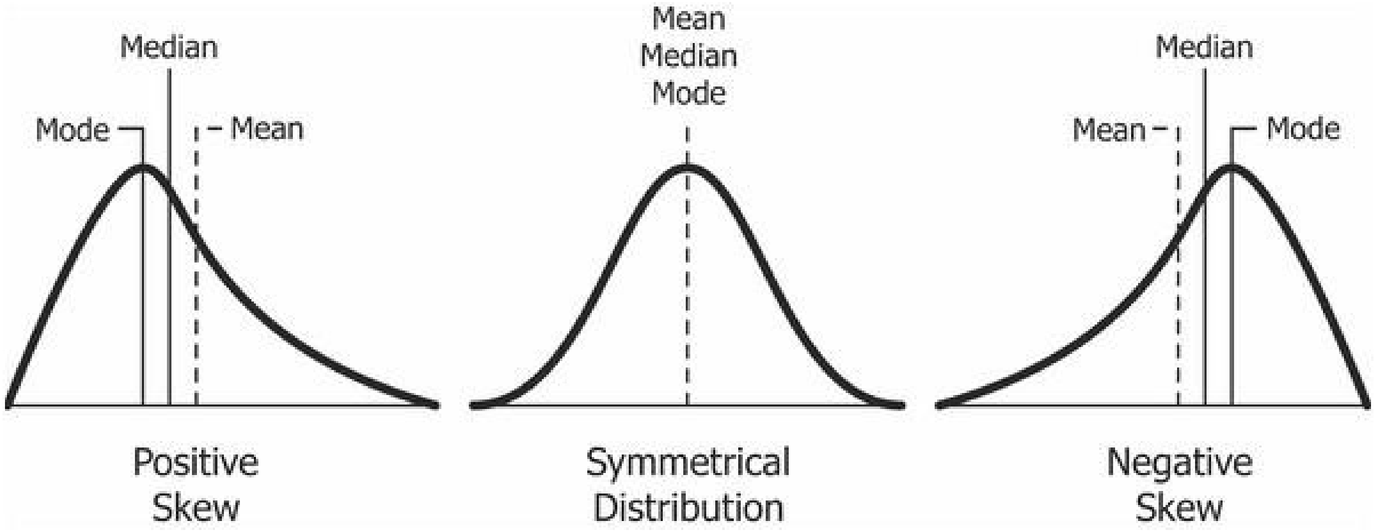
Basic Statistics

➢ **MOMENTS AND CENTRAL MOMENTS:**

Properties of Skewness:

➢A positively skewed distribution is characterized by many out lies in the upper region, or right tail. A positively skewed distribution is said to be skewed right because of its relatively long upper (right) tail.

➢A negative skewed distribution has a disproportionately large of outliers that fall within its lower (left) tail. A negative skewed distribution is said to be skewed left because of its long lower tail.



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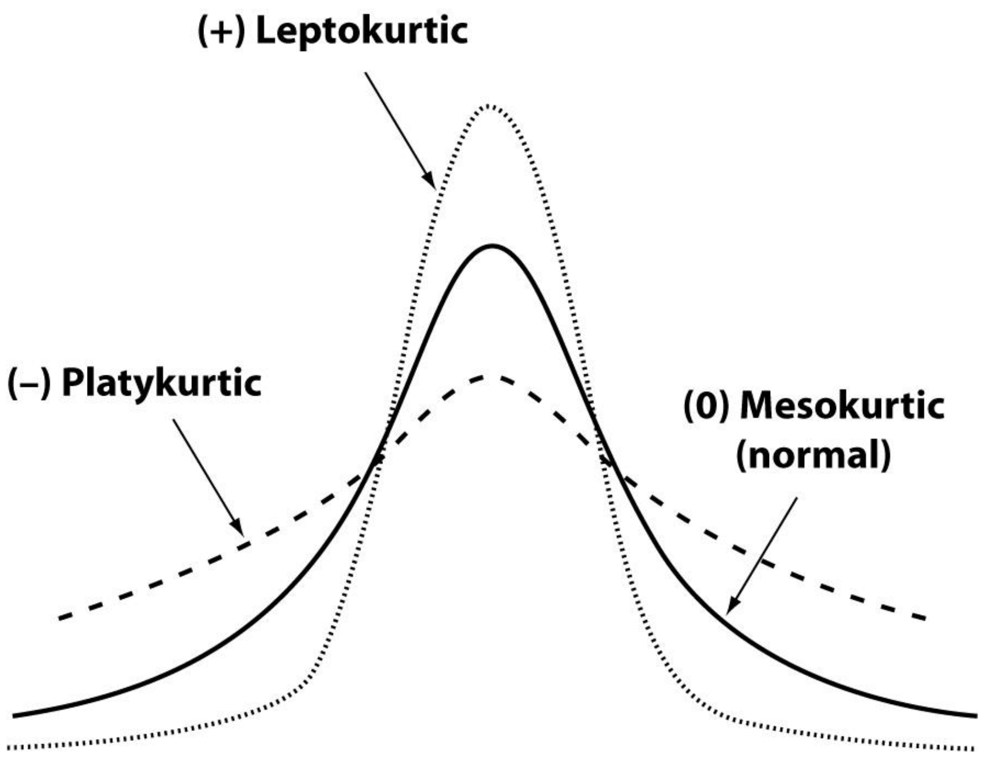
➢ **MOMENTS AND CENTRAL MOMENTS:**



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➢ MOMENTS AND CENTRAL MOMENTS:

Coskewness and Cokurtosis:

The third cross central moment is known as coskewness and the fourth cross central moment is known as cokurtosis.

suppose we are analyzing the returns data from four different stocks over a7 year time period . Although returns vary over time, the mean, variance, skewness and kurtosis of all stock returns are the same under this scenario. In addition, the covariance between returns for Stock 1 and Stock 2 is equal to the covariance between returns for Stock 3 and Stock 4.

	Stock			
Time	1	2	3	4
1	0.00%	-2.40%	-12.60%	-12.60%
2	-2.40%	12.60%	-5.30%	-5.30%
3	-12.60%	2.40%	0.00%	-2.40%
4	-5.30%	-5.30%	-2.40%	12.60%
5	2.40%	0.00%	2.40%	0.00%
6	5.30%	5.30%	5.30%	5.30%
7	12.60%	12.60%	12.60%	2.40%

- •Portfolio A = Stock 1 and Stock 2
- •Portfolio B = Stock 3 and Stock 4
- •Returns for Portfolio A and Portfolio B have the same mean and variance.
- •However, these combined return sets do not have the same skewness (i.e., the coskewness between stocks in the portfolios is different).
- •The reason for this difference is that the ranking of returns over time)e.g., from best to worst) is different for each stock, and when combined in a portfolio, these difference skew the portfolio returns distribution.

Time	A	B
1	-1.20%	-12.60%
2	-7.50%	-5.30%
3	-5.10%	-1.20%
4	-5.30%	5.10%
5	1.20%	1.20%
6	5.30%	5.30%
7	12.60%	7.50%

Most risk models choose to ignore the effects of coskewness and cokurtosis. The reason being is that as the number of variables increase, the number of coskewness and cokurtosis terms will increase rapidly, making the data much more difficult to analyze.

Basic Statistics

➢ THE BEST LINEAR UNBIASED ESTIMATOR:

➢The statistical properties of an estimator are unbiasedness, efficiency, consistency, and linearity.

➢An unbiased estimated is one for which the expected value of the estimator is equal to the parameter you are trying to estimate.

➢An unbiased estimator is also efficient if the variance of its sampling distribution is smaller than all the other unbiased estimators of the parameter you are trying to estimate. The sample mean, for example, is an unbiased deficient estimator of the population mean.

➢A consistent estimator is one for which the accuracy of the parameter estimate increase as the sample size increase. As the sample size increase, the sampling distribution bunches more closely around the population mean.

➢A point estimate is a linear estimator when it can be used as a linear function of sample data.

➢If the estimators is the best available (i.e., has the minimum variance), exhibits linearity, and is unbiased, it is said to be the best linear unbiased estimator (BLUE).

Thanks

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