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/01 Probabilities

Objective

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Describe and distinguish between continuous and discrete random variables.

Define and distinguish between the probability density function, the cumulative distribution function, andthe inverse cumulative distribution function.

Calculate the probability of an event given a discrete probability function.

Distinguish between independent and mutually exclusive events.

Define joint probability, describe a probability matrix, and calculate joint probabilities using probability matrices.

Define and calculate a conditional probability, and distinguish between conditional and unconditionalprobabilities.

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➢ RANDOM VARIABLES :

- A **random variable** is an uncertain quantity / number.
- An **outcome** is an observed valued of a random variable.
- An **event** is a single outcome or a set of outcomes.
- **Mutually exclusive events** are events that cannot happen at the same time.
- **Exhaustive events** are those that include all possible outcomes.

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➢ RANDOM VARIABLES :

➢A probability distribution describes the probabilities of all the possible outcomes for a random variable.

➢A discrete random variable is one for which the number of possible outcomes can be counted, and for each possible outcome, there is a measurable and positive probability.

➢A probability function, denoted p(x), specifies the probability that a random variable is equal to a specific value. More formally, p(x) is the probability that random variable X takes on the value x, or p(x) = P(X = x).

➢The two properties of a probability function are:

- ➢$0 \leq p(x) \leq 1$.
- ➢$\sum p(x) = 1$,the sum of the probabilities for all possible outcomes, x, for a random variable, X, equals 1.

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➢ RANDOM VARIABLES :

Example: Evaluating a probability function

Consider the following function: X = {1, 2,3,4}, $p(x) = \frac{x}{10}, else\ p(x) = 0$

Determine whether this function satisfies the conditions for a probability function.

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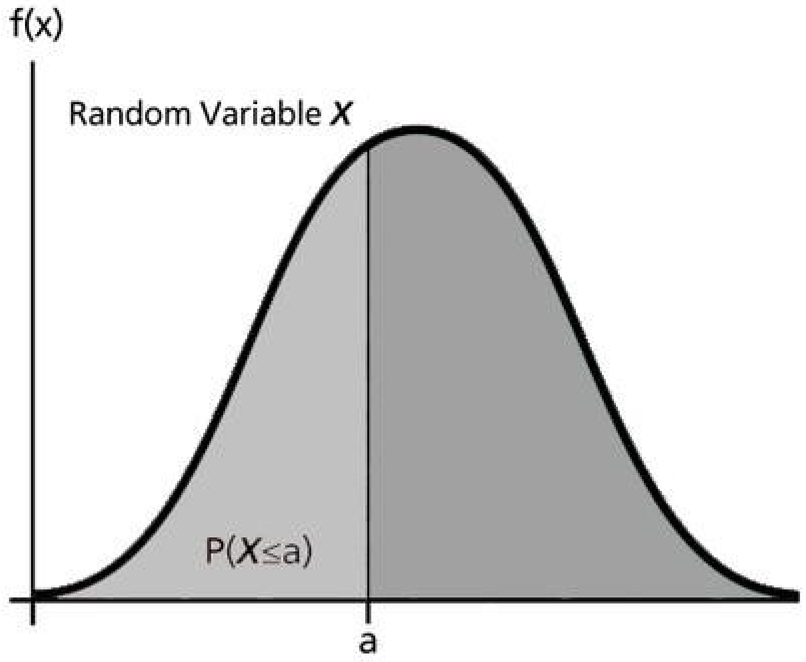
➢ DISTRIBUTION FUNCTIONS:

➢A probability density function (pdf) is a function, denoted f(x), that can be used to generate the probability that outcomes of a continuous distribution lie within a particular range of outcomes.

Probability Density Function (pdf)



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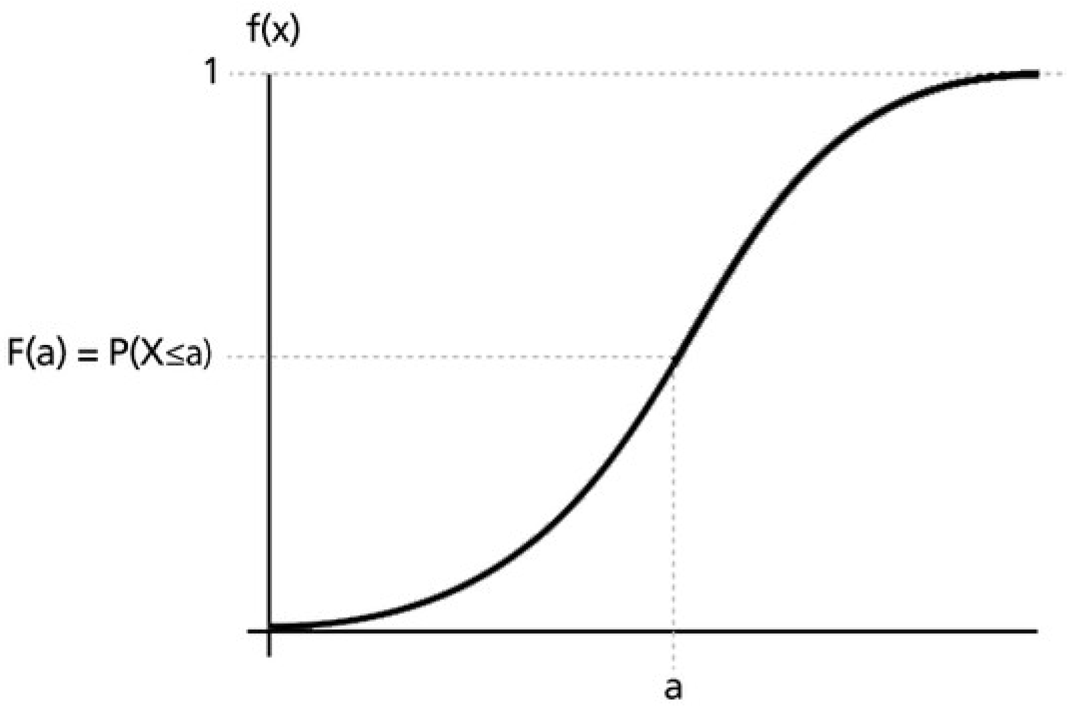
➢ DISTRIBUTION FUNCTIONS:

➢A cumulative distribution function (cdf), defines the probability that a random variable, X, takes on a value equal to or less than a specific value, x.

Cumulative Distribution Function (cdf)



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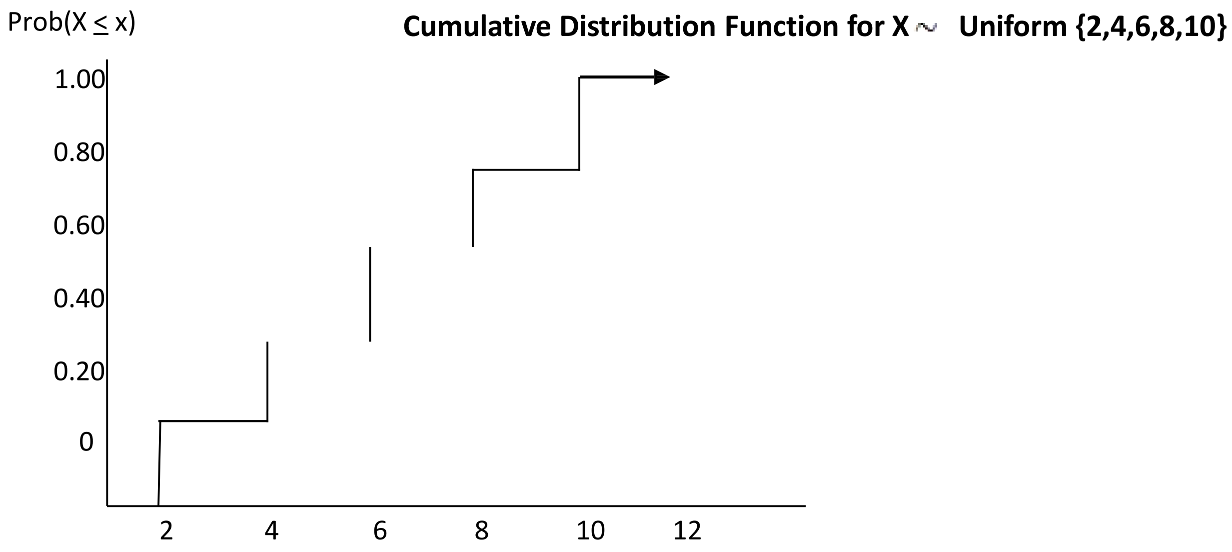
➢ DISTRIBUTION FUNCTIONS:

Discrete Probability Function

A discrete uniform variable is one for which the probability for all possible outcomes for a discrete random variable are equal.



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- **DISTRIBUTION FUNCTIONS:**

Example: Discrete uniform distribution

Determine p(6), F(6), and P(2 $\leq$ X $\leq$ 8) for the discrete uniform distribution function defined as:

x = {2,4,6,8,10}, p(x) = 0.2

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➢ CONDITIONAL PROBABILITY:

- ☐ **Unconditional probability** (i.e., marginal probability) refers to the probability of an event regardless of the past or future occurrence of other events.
- ☐ **A conditional probability** is one where the occurrence of one event affects the probability the occurrences of another event.
- ☐ The **joint probability** of two events is the probability that they will both occur.

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➢ CONDITIONAL PROBABILITY:

Multiplication Rule

For any two independent events *A and B:*

P(A and B) or P(AB)= P(A) X P(B)

For any two not independent(conditional) events *A and B:*

P(A and B) or P(AB)= P(A I B) X P(B)

Example: Multiplication rule of probability

Considerate the following information:

•P(I) = 0.4, the probability of the monetary authority increase interest rates (I) is 40%.

•P(R|I) = 0.7, the probability of a recession (R) given an increase in interest rates is 70%.

What is P(RI), the probability of a recession and an increase in interest rates?

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- **CONDITIONAL PROBABILITY:**

Addition Rule:

For any two mutually exclusive events *A and B:*

P(A or B) = P(A) + P(B)

For *any two* mutually not exclusive(conditional) *events A and B:*

P(A or B) = P(A) + P(B) - P(A and B)

Example: Addition rule of probability

Using the information in our previous interest rate and recession example and the fact that the unconditional probability of a recession, P(R), is 34%, determine the probability that either interest rates will increase or recession will occur.

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➢ INDEPENDENT AND MUTUALLY EXCLUSIVE EVENTS:

Example: Joint probability for more than two independent events (1)
What is the probability of rolling three 4s in one simultaneous toss of three dice?

Example: Joint probability for more than two independent events (2)
Using empirical probabilities, suppose we observe that the DJIA has closed higher on two thirds of all days in the past few decades. Furthermore, it has been determined that up and down days are impendent. Based on this information, compute the probability of the DJIA closing higher for five consecutive days.

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➢ **INDEPENDENT AND MUTUALLY EXCLUSIVE EVENTS:**

Example: Calculating joint probabilities using a probability matrix

Given the following incomplete probability matrix, calculate the joint probability of a normal economy and an increase in rates, and the unconditional probability of a good economy.

		Interest Rates		
		Increase	Not Increase	
Economy	Good	15%	X2	**X3**
	Normal	X1	25%	**X4**
	Poor	10%	20%	**30%**
		50%	**50%**	**100%**

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