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Multifactor Models of Risk and Return

Objective

- Describe the inputs, including factor betas, to a multifactor model.
- Calculate the expected return of an asset using a single-factor and a multifactor model.
- Describe properties of well-diversified portfolios and explain the impact of diversification on the residual risk of a portfolio.
- Explain how to construct a portfolio to hedge exposure to multiple factors.
- Describe and apply the Fama-French three factor model in estimating asset returns.

Multifactor Models of Risk and Return

➢ THE MULTIFACTOR MODEL OF RISK AND RETURN :

➢**Factor**: A variable that explains why a group of stocks have returns that tend to move together.

➢A **priced factor** is a variable which helps explain the expected returns of assets.

Q: What is the only priced factor in the CAPM?

A: The excess return on the market is the only priced factor in the CAPM.

Stocks have high expected returns if they have high sensitivity (β) with the factor.

➢A **zero cost portfolio** is a portfolio where the weights add up to zero. Usually we put zero cost portfolios in units such that the positive weights add up to 100% and the negative weights add up to -100%.

Example: You go long IBM 100% and short T-bills 100%. Zero cost portfolios are also called self financing portfolios. If the market is frictionless then a zero-cost portfolio is free. It requires no capital outlay.

➢**Multifactor Models :** The equation for a multifactor model for stock i can be expressed as follows:

Multifactor Models of Risk and Return

➢ THE MULTIFACTOR MODEL OF RISK AND RETURN :

$$R_i = E(R_i) + \beta_{i1} F_1 + \beta_{i2} F_2 + \ldots\ldots + \beta_{ik} F_k + e_i$$

- Factor beta β_{ik} is the sensitivity of the security return to a 1 unit change in the factor.
- The firm specific return e_i ,is the portion of the stocks return which is unexplained by macro factors.
- Expected value of the firm specific returns will always be equal zero.

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➢ THE MULTIFACTOR MODEL OF RISK AND RETURN :

Example: Calculate the expected return of an asset using a single-factor and a multi-factor model

1)Assume the common stock of Health Care Inc. (HCI) is examined with a single-factor model, using unexpected percent changes in GDP as the single factor. Assume the following data is provided:

Expected return for HCI = 10%

GDP factor beta = 2.00

Expected GDP growth = 3%

Suppose new macroeconomic information indicates that GDP growth will equal 4% rather than the original consensus forecast of 3%. Also assume there's no new information regarding firm-specific events. Calculate the revised expected return for HCI using a single-factor model.

Multifactor Models of Risk and Return

➢ THE MULTIFACTOR MODEL OF RISK AND RETURN :

Example: Calculate the expected return of an asset using a single-factor and a multi-factor model

2) Assume the common stock of Health Care Inc. (HCI) is examined using a multifactor model, based on two factors: unexpected percent change in GDP and unexpected percent change in consumer sentiment. Assume the following data is provided:

Expected return for HCI = 10%

GDP factor beta = 2.00

Consumer sentiment (CS) factor beta = 1.50

Expected growth in GDP = 3%

Expected growth in consumer sentiment = 1%

Suppose new macroeconomic information indicates that GDP will grow 4% rather than 3%, and that consumer sentiment will grow 3% rather than 1%. Calculate the return for HCI using 2-factor model.

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➢ THE LAW OF ONE PRICE AND ARBITRAGE OPPORTUNITES:

According to **Law of One Price**, identical assets selling in different locations should be priced identically in the different locations. The action of buying an asset in the cheaper market and simultaneously selling that asset in the more expensive market is called **arbitrage**.

Example:
Assume a 2-factor model is used to examine the returns for two assets, A and B. Also, assume the two factors are unexpected percentage changes in GDP and consumer sentiment (CS).

$$R_A = E(R_A) + \beta_{A,GDP} F_{GDP} + \beta_{A,CS} F_{CS} + e_A$$

$$R_B = E(R_B) + \beta_{B,GDP} F_{GDP} + \beta_{B,CS} F_{CS} + e_B$$

Additional data :

$E(R_A)$	=0.12
$\beta_{A,GDP}$ $\beta_{B,GDP}$	=0.10
$\beta_{A,CS}$	=2
$\beta_{B,CS}$	=1.5

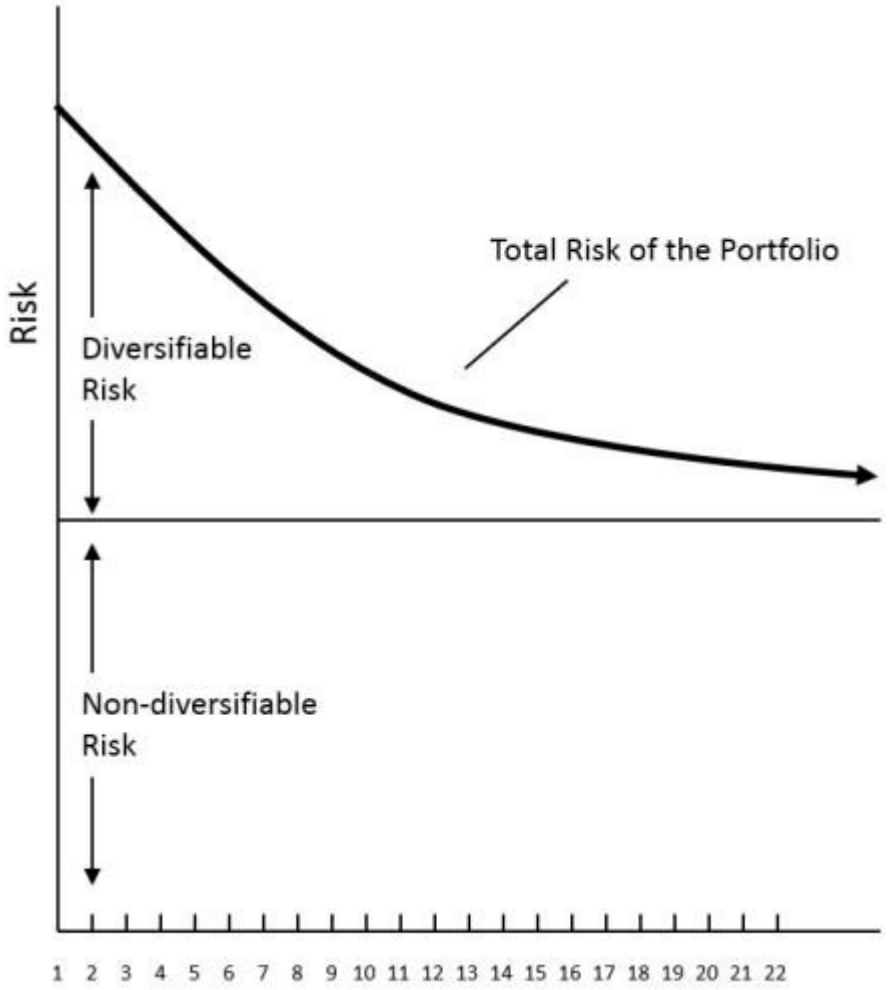
The expected return and factor betas for the long-short hedge portfolio, H, are calculated as follows:

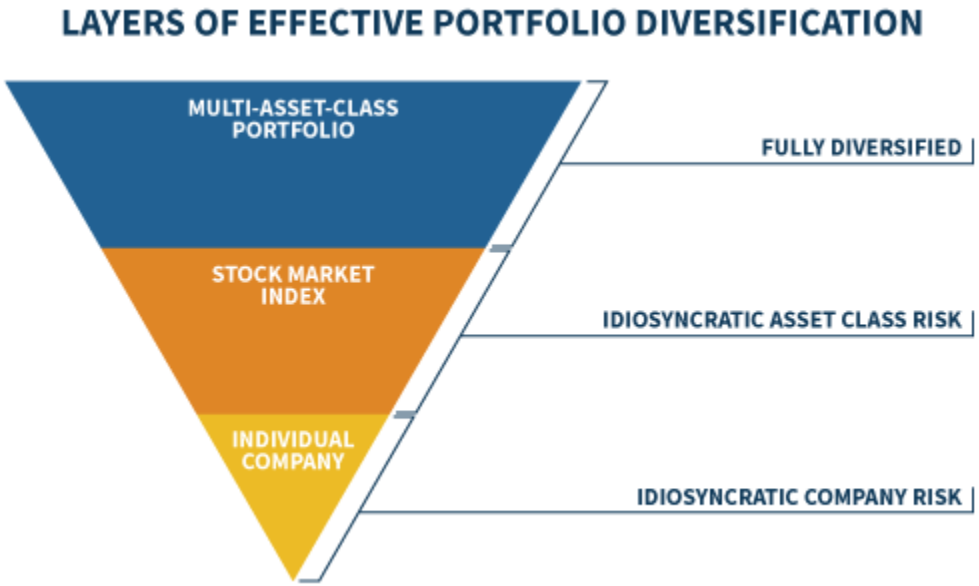
$E(R_H) = E(R_A) - E(R_B) =$ 0.02
$\beta_{H,GDP} = \beta_{A,GDP} - \beta_{B,GDP} =$ 0
$\beta_{H,CS} = \beta_{A,CS} - \beta_{B,CS} =$ 0

- Therefore, the net investment in Portfolio H is zero (invest $1 in Asset A for every $1 cold short in Asset B).
- The net position for Portfolio H is riskless (i.e., factor betas are zero), but the expected profit from Portfolio H is positive (equal to 2%).
- These condition constitute an arbitrage situation.

Multifactor Models of Risk and Return

➢ **WELL-DIVERSIFIED PORTFOLIOS :**



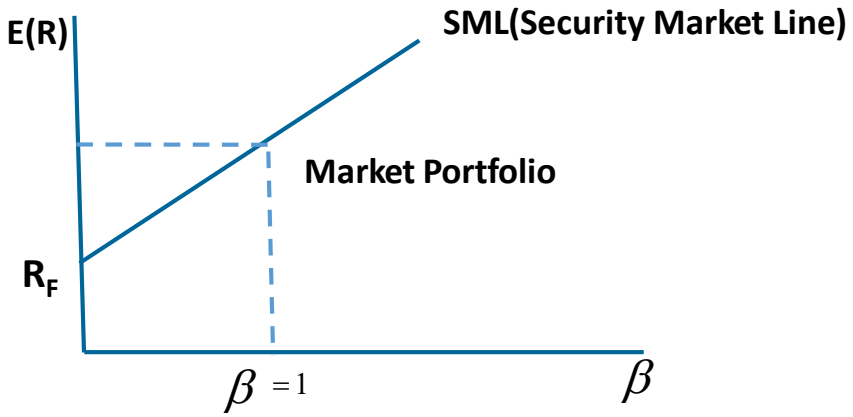


•Because nonsystematic (diversifiable) risk can be avoided (without cost) by efficient diversification, there is no added expected return for bearing nonsystematic risk.

•This result is the crucial point of the capital asset pricing model (CAPM) and is expressed in the equation of the security market line (SML).

Multifactor Models of Risk and Return

➢ THE SINGLE-FACTORS SECURITY MARKET LINE:



- The implication of the single-factor security market line is that, when no arbitrage opportunities exist, all well-diversified portfolios must lie on the line.
- The equation for the line (for any well-diversified Portfolio P) is :

$$E(R_P) = R_F + \beta_P[E(R_M) - R_F$$

- The key difference between CAPM & single-factor security market line is that :
- The CAPM relies on the existence of the mean-variance efficient frontier, and consists of all marketable assets).
- In contrast, the equation for the single-factor security market line merely relies on the assumptions that security return can be explained by a single-factor model, that well-diversified portfolio can be created, and that no arbitrage opportunities exist.

Multifactor Models of Risk and Return

➢ THE SINGLE-FACTORS SECURITY MARKET LINE:

HEDGING EXPOSURES TO MULTIPLE FACTORS:

Consider an investor who manages a portfolio with the following factor betas:
GDP beta = 0.50
Consumer sentiment beta = 0.30
Assume the investor wishes to hedge away GDP factor risk, yet maintain the 0.30 exposure to consumer sentiment.
To do so, the investor should combine the original portfolio with a 50% short position in the GDP factor portfolio.
Assume the investor might want to hedge away consumer sentiment (CS) factor risk, yet maintain the 0.50 exposure to GDP.
To do so, the investor should combine the original portfolio with a 30% short position in the CS factor portfolio.
Assume if the investor wants to hedge away both factor risks?
To do so, first, the investor should from a portfolio invested 50% in the GDP factor portfolio and 30% in the CS factor portfolio (and the remaining 20% in the risk-free asset).

Multifactor Models of Risk and Return

➢ THE ARBITRAGE PRICING THEORY:

The arbitrage pricing theory (APT) described expected returns as a linear function of exposures to common (i.e., macro economic) risk factors: $E(R_i) = R_F + \beta_{i1} RP_1 + \beta_{i2} RP_2 + \ldots\ldots + \beta_{ik} RP_k$

Step 1
- Create factor portfolios.
- Repeat the process for all k factors in the multifactor model so that a factor portfolio is derived for each of the risk factors.

Step 2
- Derive returns for each factor portfolio.

Step 3
- Calculate risk premiums for each factor portfolio.

The assumptions underlying the APT model are as follows :

- Returns follow a k-factor process
- Well-diversified portfolios can be formed.
- No arbitrage opportunities exist.

Given the derivation of the risk premiums, the APT model can be rewritten as :

$$E(R_i) = R_F + \beta_{i1}[E(R_1) - R_F] + \beta_{i2}[E(R_2) - R_F] + \ldots\ldots + \beta_{ik}[E(R_k) - R_F]$$

Multifactor Models of Risk and Return

➢ THE ARBITRAGE PRICING THEORY:

Example : Compute expected return using the APT model

Assume the following data for Asset Z:

Risk-free rate = 5%, GDP factor beta = 0.50, Consumer sentiment factor beta = 0.30, GDP risk premium = 4%, Consumer sentiment risk premium = 3%

Calculate the expected return for Asset Z using a 2-factor APT model.

Multifactor Models of Risk and Return

➢ The Fama- French Three-Factor Model:

A major weakness of the APT is that it offers no guidance as to the identification of the appropriate risk factors. Professions Eugene Fama and Kenneth French, in contrast to the APT model, they identified two additional factors other than market return factor.

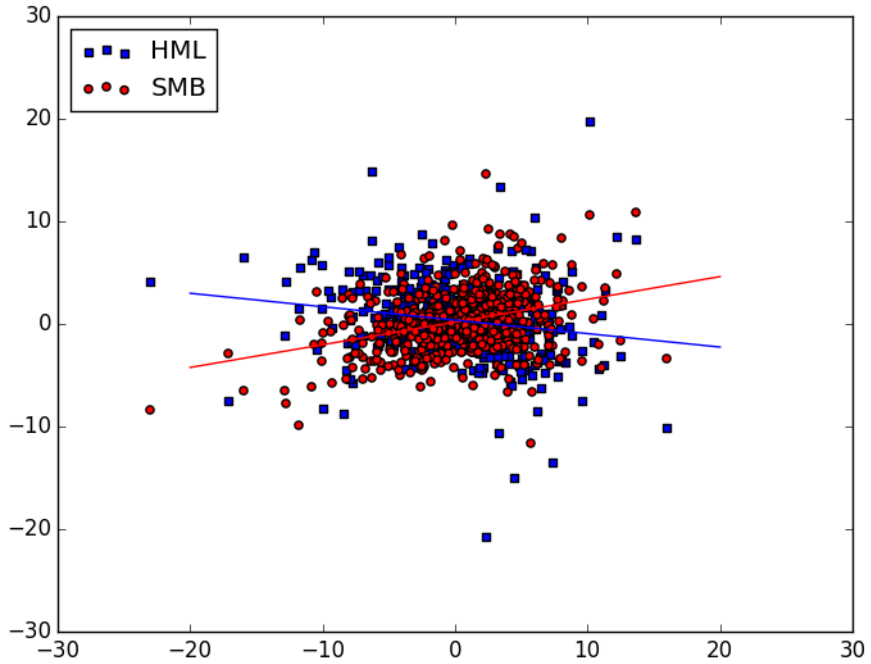
SMB (small minus big) is the firm size factor equal to the difference in returns between portfolios of small and big firms ($R_S - R_B$).

HML (high minus low) is the book-to-market (i.e., book value per share divided by stock price) factor equal to the difference in return between portfolios of high and low book-to-market firms ($R_H - R_1$).

The equation for the Fama-French three- factor model is :

$$R_i - R_F = \alpha_i + \beta_{i,M}(R_M - R_F) + \beta_{i,SMB}SMB + \beta_{i,HML}HML + e_i$$

The intercept term (i.e., alpha) equals the abnormal performance of the asset after controlling for its exposures to the market, firm size, and book-to- market factors.



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