/01 资本资产定价理论（Capital Asset Pricing Model）

Objective

- Understand the derivation and components of the CAPM.
- Describe the assumptions underlying the CAPM.
- Interpret the capital market line.
- Apply the CAPM in calculating the expected return on an asset.
- Interpret beta and calculate the beta of a single asset or portfolio.

DELINEATING EFFICIENT PORTFOLIOS

EXPECTED RETURN AND-VOLATILITY OF A TWO-ASSET PORTEFOLIO:

$E(R_P) = w_1E(R_1) + w_2E(R_2)$

Where:

$E(R_P)$ = expected return on Portfolio P

w_i = proportion (weight) of the portfolio allocated to Asset i

$E(R_i)$ = expected return on Asset i

The weights (w_1 and w_2) must sum to 100% for a two-asset portfolio.

The correlation coefficient of a two-asset portfolio equals :

$$\rho_{1,2} = \frac{Cov_{1,2}}{\sigma_1\sigma_2}$$

Example : Expected return and volatility for a two-asset portfolio

Using the information in the following figure, calculate the expected return and standard deviation of the two-asset portfolio.

Characteristics for a Two-Stock Portfolio

	Caffeine Plus	*Sparkin'*
Amount invested	\$40,000	\$60,000
Expected return	11%	25%
Standard deviation	15%	20%
Correlation	0.30	

The variance of a two-asset portfolio equals :

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov_{1,2}$$

σ_P^2 = variance of the returns for Portfolio P

σ_1^2 = variance of the returns for Asset 1

σ_2^2 = variance of the returns for Asset 2

W_i = weight allocated to Asset I in portfolio

$Cov_{1,2}$ = covariance between the returns of the two assets

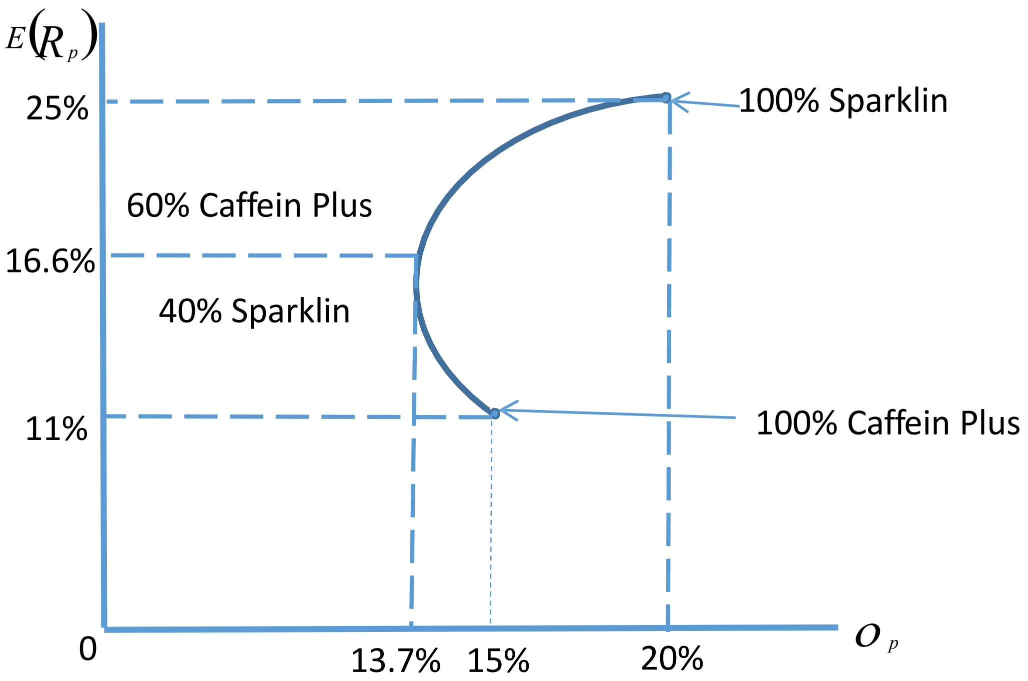
- The covariance, $Cov_{1,2}$ measures the strength of the relationship between the returns earned on assets 1 and 2.
- The covariance's is unbounded (ranges from negative infinity to positive infinity); therefore, it is not a very useful measure of the strange of the relationship between two asset's returns.
- The correlation coefficient is bounded by -1 to +1.
- -1 indicates perfect negative correlation, whereas +1 indicates perfect positive correlation.

DELINEATING EFFICIENT PORTFOLIOS

THE PORTFOLIO POSSIBILITY CURVE:

Figure 1 : Portfolio Returns for Various Weights of Two Assets

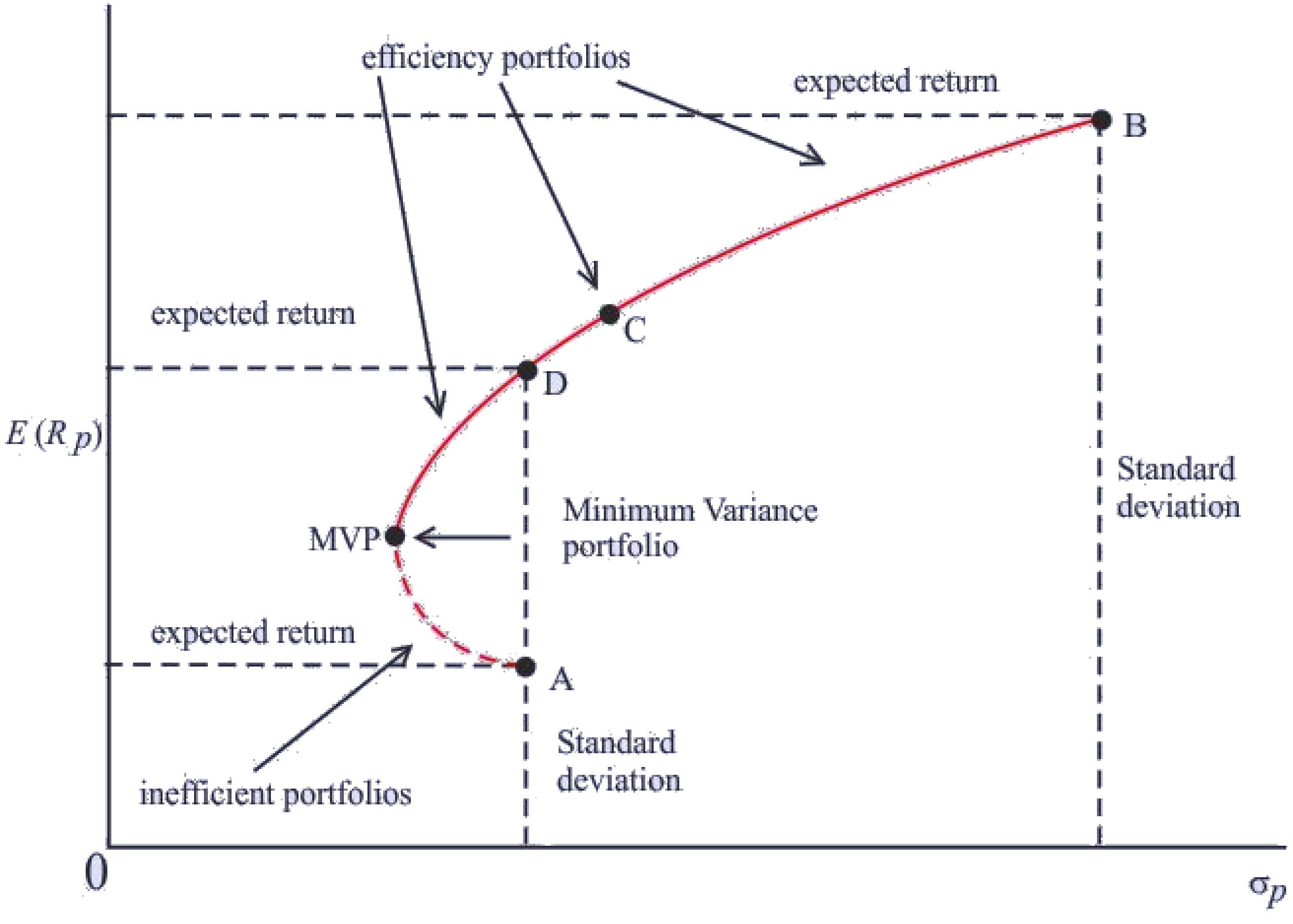
w Caffeine Plus w Sparklin'	100% 0%	80% 20%	60% 40%	40% 60%	20% 80%	0% 100%
$\hat{R}_p$	11.00%	13.80%	16.60%	19.40%	22.20%	25.00%
σ_p	15.00%	13.74%	13.72%	14.94%	17.10%	20.00%



DELINEATING EFFICIENT PORTFOLIOS

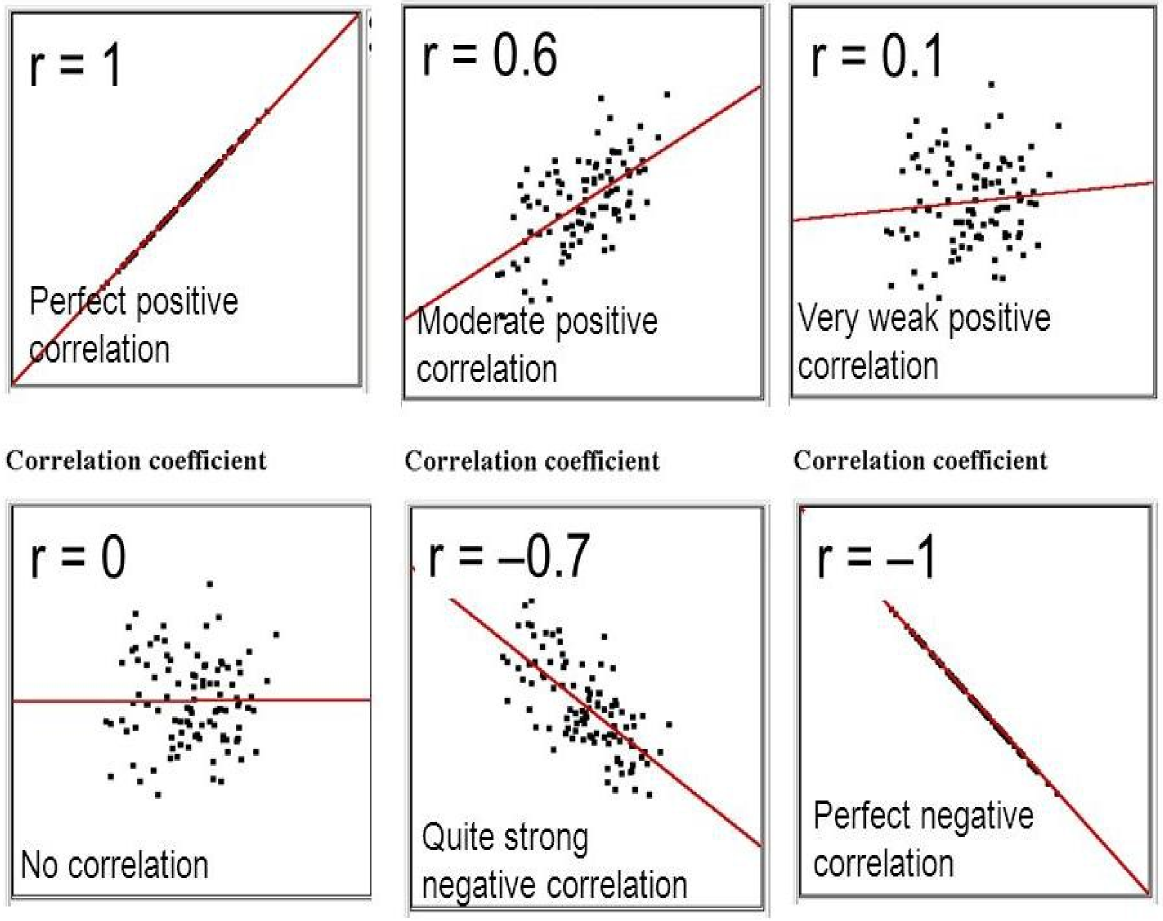
MINIMUM VARIANCE PORTFOLIO:

- **The minimum variance portfolio** is the portfolio with the smallest variance among all possible portfolio
- on a portfolio possibilities curve.



DELINEATING EFFICIENT PORTFOLIOS

CORRELATION AND PORTFOLIO DIVERSIFICATION:



Portfolio Standard Deviation

- Perfect Positive Correlation: $\sigma_p = w_1\sigma_1 + w_2\sigma_2$
- Perfect Negative Correlation: $\sigma_p = \pm(w_1\sigma_1 + w_2\sigma_2)$
- Zero Correlation: $\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2}$

DELINEATING EFFICIENT PORTFOLIOS

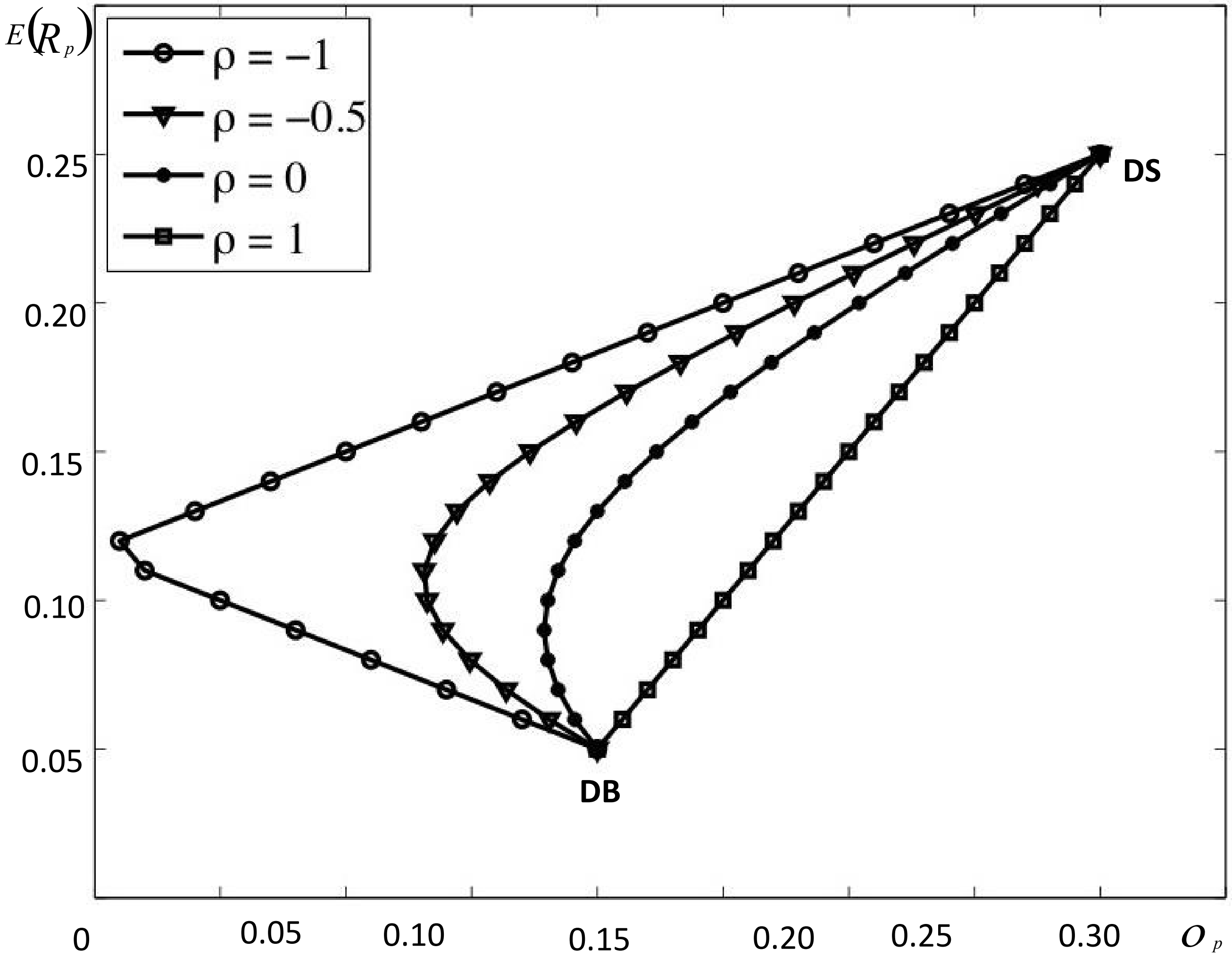
CORRELATION AND PORTFOLIO DIVERSIFICATION:

An Example of Correlation and Portfolio Diversification

	Expected Return	Standard Deviation
Domestic Stocks (DS)	0.20	0.30
Domestic Bonds (DB)	0.10	0.15

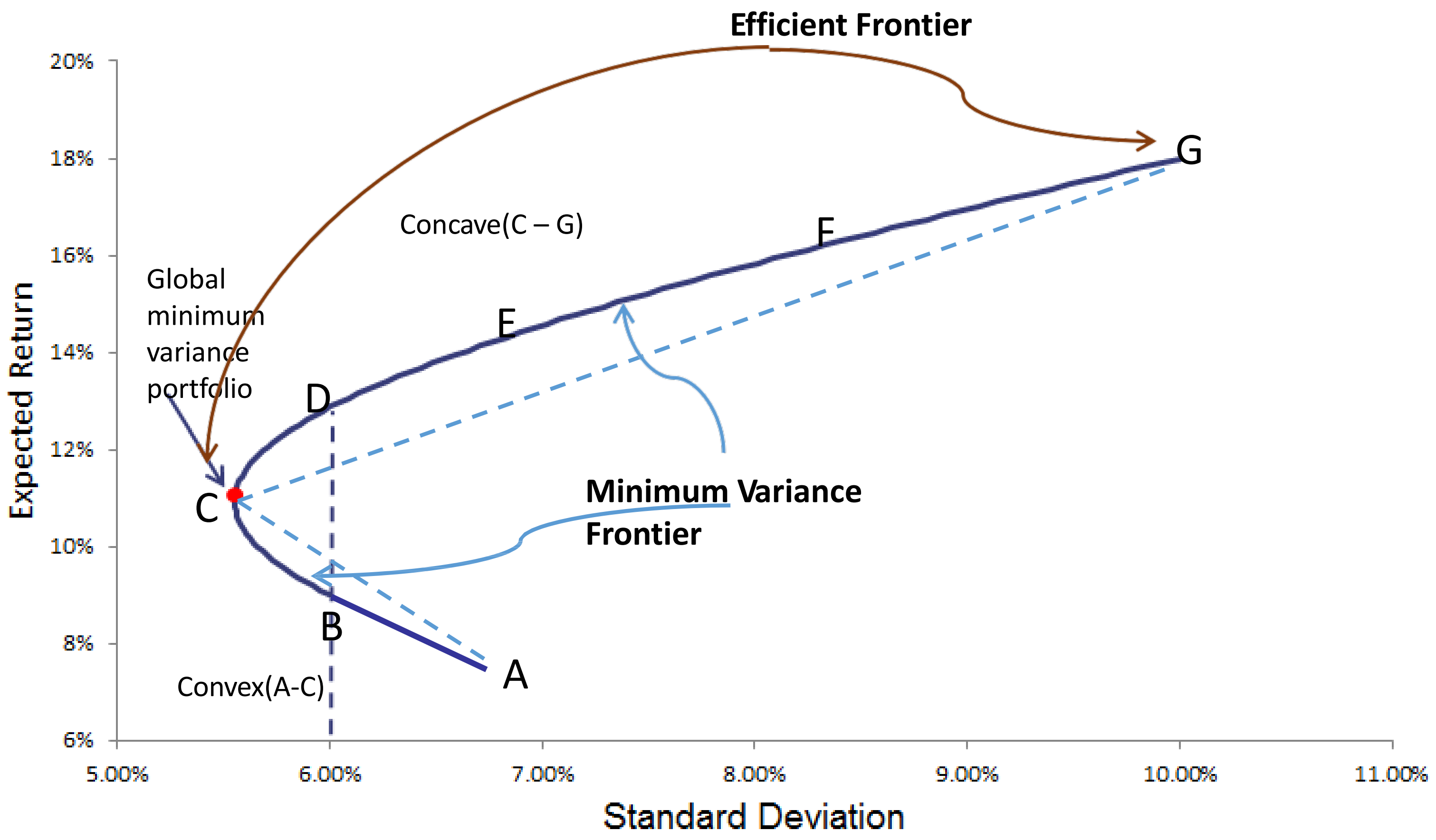
Expected Return / Standard Deviation Combination for Various Allocations

DS % Allocation	*DB % Allocation*	*$E(R_p)$*	Correlations			
			ρ = 1	*ρ = 0.5*	*ρ = 0*	*ρ = 1*
100.00	0.00	0.200	0.300	0.300	0.300	0.300
66.67	33.33	0.167	0.250	0.229	0.206	0.150
50.00	50.00	0.150	0.225	0.198	0.168	0.075
33.33	66.67	0.133	0.200	0.173	0.141	0.000
0.00	100.00	0.100	0.150	0.150	0.150	0.150



DELINEATING EFFICIENT PORTFOLIOS

THE SHAPE OF THE PORTFOLIO POSSIBILITIES CURVE:



DELINEATING EFFICIENT PORTFOLIOS

THE EFFICIENT FRONTIER:

Short Sales and the Efficient Frontier:

- ✓When short sales are allowed, the shape of the efficient changes.
- ✓When allowing for short sales, the efficient expands up and to the right.
- ✓By shorting it is possible to create higher return and higher volatility portfolio combination that would not be possible otherwise.
- ✓Theoretically, with no limitations on shorting, it that would not be possible to construct a portfolio with infinite return.

Portfolio Returns for Various Weights of Two Assets (w/ Short Sales)

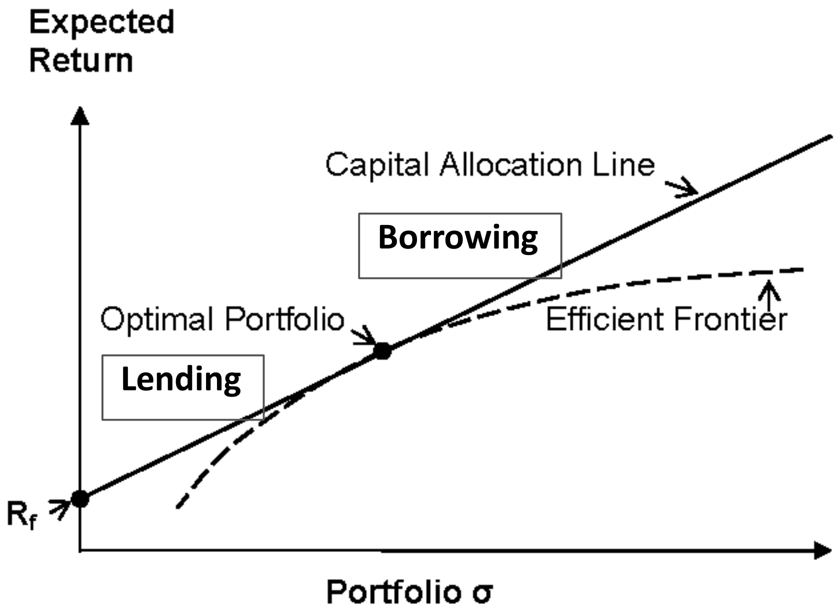
w Caffeine Plus	100%	80%	60%	40%	20%	0%	-20%	-40%	-60%	-80%	-100%
wSparklin'	0%	20%	40%	60%	80%	100%	120%	140%	160%	180%	200%
$\hat{R}_p$	11.00%	13.80%	16.60%	19.40%	22.20%	25.00%	27.80%	30.60%	33.40%	36.20%	39.00%
σ_p	15.00%	13.74%	13.72%	14.94%	17.10%	20.00%	23.28%	26.82%	30.53%	31.36%	38.28%

DELINEATING EFFICIENT PORTFOLIOS

THE EFFICIENT FRONTIER:

Combining the Risk-Free Rate with the Efficient Frontier:

- Assume a portfolios comprising the risk- free asset F, and a risky portfolio P and the portfolio P lies on the efficient frontier of risky assets.
- Various combinations (weightings) of Portfolio P and the risk-free asset can be created.
- By adding the risk-free asset to the investment mix, a very important property emerges: the shape of the efficient
- frontier changes from a curve to a line.



Capital Allocation Line ≈ Capital Market Line

Standard deviation of investment C is the weighted standard deviation of risky portfolio P.

$$\sigma_c = w_p \sigma_p$$

Expected return of investment C is

$$E(R_C) = R_F + \left(\frac{E(R_P) - R_F}{\sigma_P}\right)\sigma_C$$

THE STANDARD CAPITAL ASSET PRICING MODEL

THE CAPITAL ASSET PRICING MODEL (CAPM):

- The capital asset pricing model (CAPM), derived by Sharpe, Lintner, and Mossin.
- The CAPM provides a ways to calculate an asset's expected return (or "required" return) based on its level of
- systematic (or market-related) risk, as measured by the asset's beta.
- The CAPM calculation for expected return of asset i :-

$$E(R_i) = R_F + \beta[E(R_M - R_F)]$$

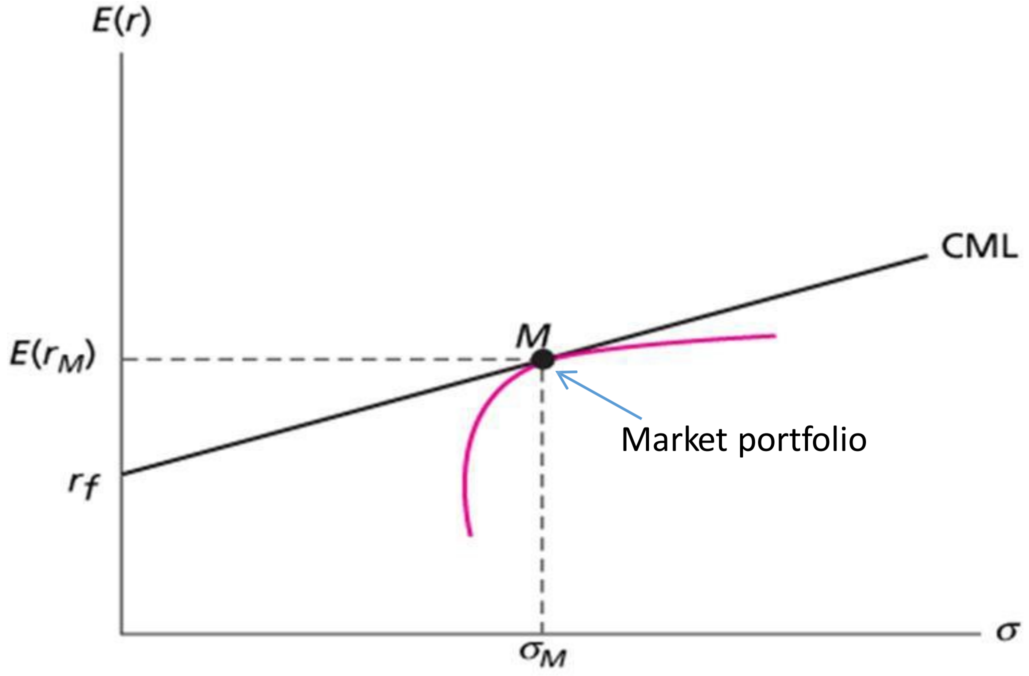
CAPM Assumptions:-

➢Investors face no transaction costs when trading assets.
➢Assets are infinitely divisible.
➢There are no taxes.
➢Investors are price takers whose individual buy and sell decisions have no effect on asset prices.
➢Investors' utility functions are based solely on expected portfolio return and risk.
➢Unlimited short-selling is allowed.
➢Investors can borrow and lend unlimited amounts at the risk-free rate.
➢Investors are only concerned about returns and risk over a single-period, and the single period is the same for all investors.
➢All investors have the same forecasts of expected returns, variances, and covariance.
➢All assets are marketable, including human capital.

THE STANDARD CAPITAL ASSET PRICING MODEL

THE CAPITAL MARKET LINE (CML) :

- ➢Under the assumptions of the CML, all investors agree on the exact composition of the optimal risky portfolio.
- ➢This universally agreed upon optimal risky portfolio is called the market portfolio, M and it is defined as the portfolio of all marketable assets weighted in proportion to their relative market values.
- ➢All investors will make optimal investment decisions by allocating between the risk-free asset and the market portfolio.



CML Equation: $$E(R_P) = R_F + \left(\frac{E(R_M) - R_F}{\sigma_M} \right) \sigma_P$$

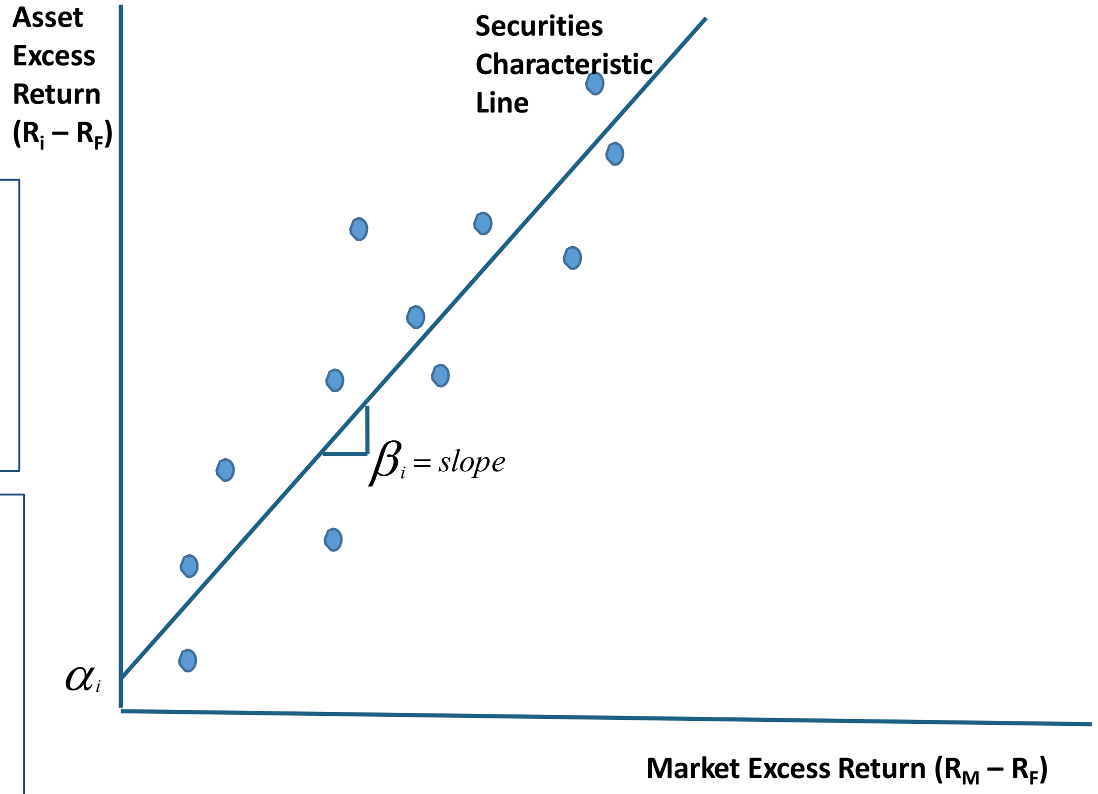
Sharp ratio: $$\left(\frac{E(R_M) - R_F}{\sigma_M} \right)$$ ← Market price of risk

THE STANDARD CAPITAL ASSET PRICING MODEL

BETA:

- The sensitivity of an asset’s return to the market return is referred to as the asset’s beta.
- Beta is a standardized measure of the covariance of the covariance of the asset’s return with the market return.
- Beta can be calculated as follows: $\beta_i = \frac{Cov_{i,M}}{\sigma_M^2}$
- Beta can also calculated as: $\beta_i = \rho_{i,M} \frac{\sigma_i}{\sigma_M}$

Regression of Asset Excess Returns against Market Asset Returns



Example : Calculating an asset’s beta

The standard deviation of the market return is estimated as 20%.

- If Asset A’s standard deviation is 30% and its correlation of returns with the market index is 0.8, what is Asset A’s beta?
- If the covariance of Asset A’s returns with the return on the market index is 0.048, what is the beta of Asset A?

Example : Calculating portfolio beta

Consider the following individual asset weights and betas for a 4-asset portfolio.

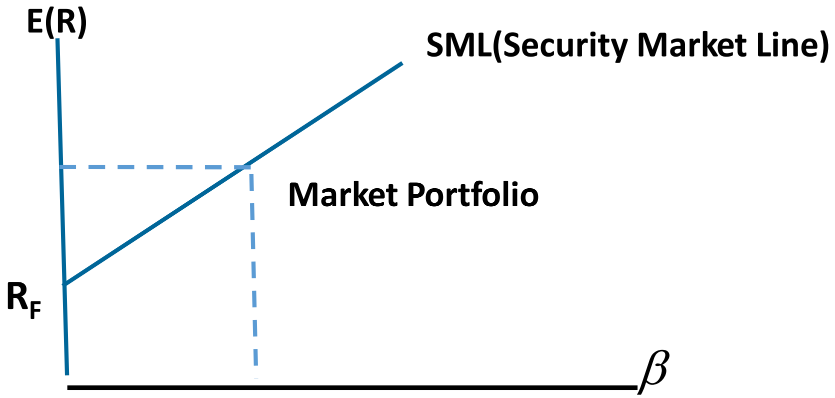
Asset	*Portfolio Weight*	*Beta*
1	25%	1.2
2	15%	1.8
3	35%	0.9
4	25%	1.4

Calculate the beta of this 4-asset portfolio.

THE STANDARD CAPITAL ASSET PRICING MODEL

DERIVING THE CAPM

- ➢Beta identifies the appropriate level of risk for which an investor should be compensated.
- ➢As a portfolio becomes more diversified, idiosyncratic risk (i.e., unsystematic risk or asset- specific risk) in the portfolio becomes less of an issue as only systematic risk remains.
- ➢Since diversification is costless and systematic risk is the only remaining risk in a diversified portfolio, an investor should only be compensated for systematic risk (or beta) exposure.
- ➢Hence, all assets with the same beta should earn the same return.
- ➢Since portfolio beta is the weighted average of the individual betas and expected portfolio return is a weighted average of the individual expected return, hence the portfolio expected return is a linear function of beta.
- ➢Expected return of asset i is: $E(R_i) = R_F + \beta_i[E(R_M) - R_F]$
- ➢Where $[E(R_M) - R_F]$ is called risk premium.
- ➢Market beta is always equal one.



THE STANDARD CAPITAL ASSET PRICING MODEL

CALCULATING EXPECTED RETURN USING THE CAPM:

Example: Expected return on a stock

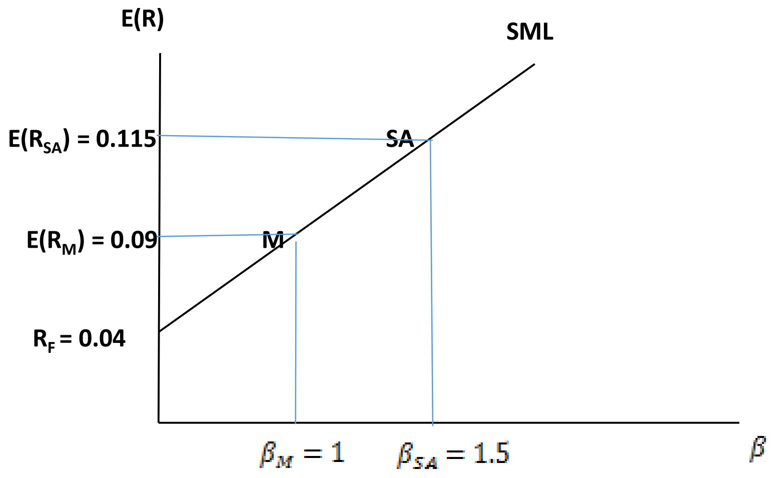
Assume you are assigned the task of evaluation the stock of Sky-Air, Inc. To evaluate the stock, you calculate return using the CAPM. The following information is available :

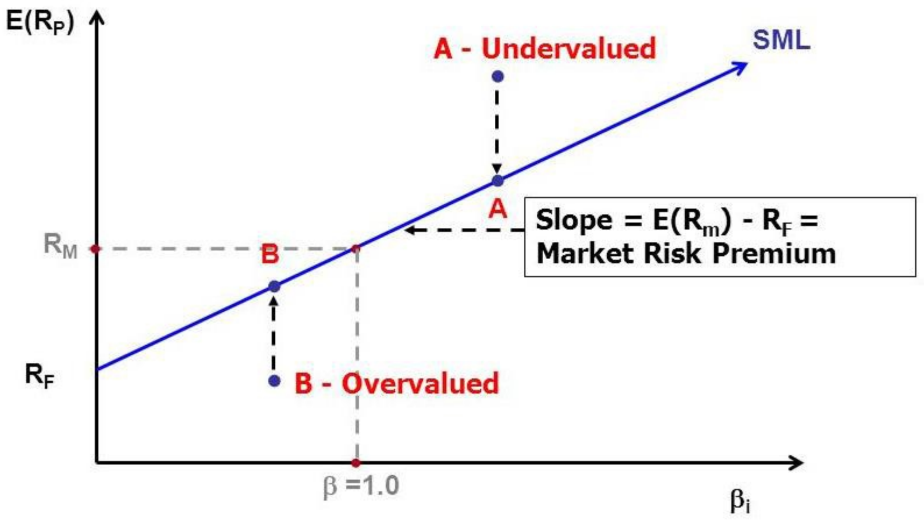
Expected market risk premium	5%
Risks-free rate	4%
Sky-Air beta	1.5

Using CAPM, Calculate and interpret the expected return for Sky-Air.

Example : Using CAPM to calculate the expected market returns

A stock has a beta of 0.75 and an expected return of 13%. The risk-free rate is 4%. Calculate the market risk premium and the expected return on the market portfolio.





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